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A PORTABLE Computer Program for Designing and  
Implementing Recursive Digital Filters

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and Development Command  
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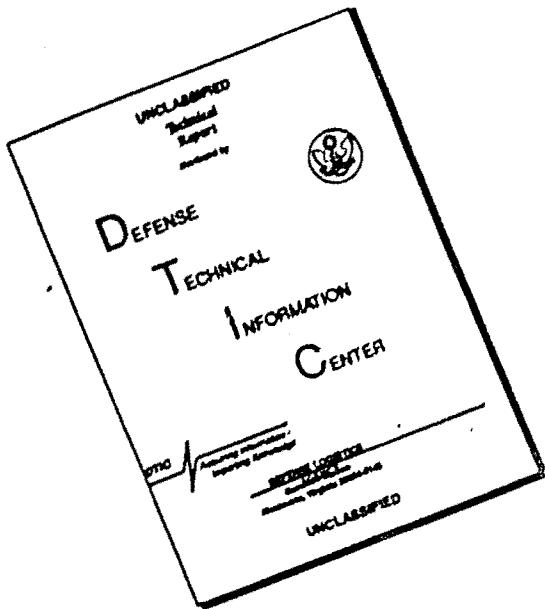
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band-pass, and band-stop filters from the low-pass designs. The development of the computer program is detailed along with all the necessary equations. A complete listing of the code also is provided.

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FOREWORD

This report resulted from a joint project assigned to the authors for a course on digital signal processing offered at the George Washington University, Washington, DC. The authors thank Professor N. Kyriakopoulos for his guidance during the project.

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## 1. INTRODUCTION

The purpose of this project is to develop a computer program that will design and implement a recursive digital filter to meet user specifications given as inputs to the program. The filter type is selected from three well-known analog filter approximations: (1) Butterworth, maximally flat; (2) Chebyshev, equiripple in the passband; and (3) elliptic (or Cauer), equiripple in both the passband and the stop band. For a low-pass filter, for example, the user specifies the filter type, the filter order, the cutoff frequency, and the sampling rate. High-pass, band-pass, and band-stop filters also may be chosen for design.

The program calculates the poles and the zeros of the low-pass filter in the analog  $s$  domain. The bilinear transformation technique is then used to determine the coefficients of the transfer function in the  $z$  domain. If a high-pass, band-pass, or band-stop filter is required, then the coefficients are determined by a transformation in the  $z$  domain of the prototype low-pass digital filter whose coefficients have been determined from an analog low-pass filter using the bilinear transformation.

After determining the coefficients of the digital filter transfer function, a plot of the magnitude and the phase may be obtained if desired by the user. A list of the coefficients also is printed. Another option is to implement the filter as a system of linear difference equations and to process input data consisting of sample values at any given sample rate through the filter. The filtered output data points are then printed as output.

In summary, the program performs the following:

- a. Calculates and prints the coefficients of the desired digital filter.
- b. Plots the magnitude and phase responses.
- c. Implements the digital filter and processes data through the filter.
- d. Outputs the filtered data.

## 2. APPROACH

There are two general types of digital filters, recursive and non-recursive. The output response of a nonrecursive filter is a function

of only the present and past values of the input excitation. A recursive filter is one in which the present output response is a function of the present and past values of the input, as well as past values of the output.

As in analog filters, the approximation step in the design of digital filters is the process whereby a realizable transfer function satisfying prescribed specifications is obtained. To be realizable as a recursive filter, a transfer function must satisfy the following constraints.

- a. It must be a rational function of  $z$  with real coefficients.
- b. Its poles must lie within the unit circle of the  $z$  plane.
- c. The degree of the numerator polynomial must be equal to or less than that of the denominator polynomial.

Recursive filter approximations can be obtained from analog filter approximations by using the following methods, all of which satisfy the above constraints:

Invariant-impulse-response method  
Matched  $z$  transformation  
Bilinear transformation

Each method has certain advantages and disadvantages.

### 2.1 Invariant-Impulse-Response Method

Given an analog filter transfer function,  $H_A(s)$ , the invariant-impulse-response method is implemented as follows:

- a. Obtain the impulse response of analog filter  $H_A(t)$ .
- b. Replace  $t$  by  $nT$  in  $H_A(t)$ .
- c. Form the  $z$  transform of  $H_A(nT)$ . This gives  $H_D(z)$ , the digital filter transfer function.  $T$  is the sample period =  $1/f_s$ .

This method gives good results if  $H_A(j\omega) \approx 0$  for  $\omega > \omega_s/2$ . However, aliasing errors tend to restrict this method to the design of all pole filters.

### 2.2 Matched $z$ Transformation

Given continuous-time transfer function

$$H_A(s) = \frac{H_0 \prod_{i=1}^M (s - s_i)}{\prod_{i=1}^N (s - p_i)},$$

a corresponding digital transfer function can be formed as

$$H_D(z) = (z + 1)^L \frac{H_0 \prod_{i=1}^M (z - e^{s_i T})}{\prod_{i=1}^N (z - e^{p_i T})},$$

where  $L$  is an integer equal to the number of zeros at  $s = \infty$  in  $H_A(s)$ . This method gives reasonable results for high-pass and band-stop filters, although it tends to distort the passband ripple in Chebyshev and elliptic filters. For low-pass and band-pass filters, better approximations can be obtained by using the modified invariant-impulse-response method.<sup>1</sup>

### 2.3 Bilinear Transformation

The bilinear transformation method yields a digital filter with approximately the same time-domain response as the original analog filter for any excitation.

$$H_D(z) = H_A(s) \Bigg|_{s = \frac{2}{T} \frac{z - 1}{z + 1}}$$

The bilinear transformation maps these:

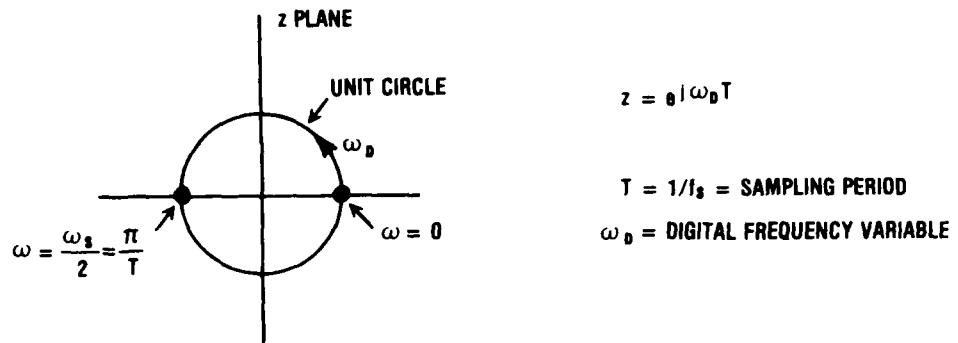
- a. The open right-half  $s$  plane onto the region exterior to the unit circle  $|z| = 1$  of the  $z$  plane
- b. The  $j$  axis of the  $s$  plane onto the unit circle  $|z| = 1$
- c. The open left-half  $s$  plane onto the interior of the unit circle

From property c it follows that a stable analog filter yields a stable digital filter and, since the transformation has real coefficients,  $H_D(z)$  has real coefficients.

---

<sup>1</sup>A. Antoniou, *Digital Filters: Analysis and Design*, McGraw-Hill Book Co., Inc., New York (1979).

Digital filters obtained by the bilinear transformation do not suffer from the effects of aliasing. However, a nonlinear frequency distortion is introduced because the transformation maps the entire  $j\omega$  axis onto the unit circle (fig. 1).



THE ANALOG FREQUENCY VARIABLE,  $\omega_A$ , IS RELATED TO THE DIGITAL FREQUENCY VARIABLE,  $\omega_D$ , THROUGH THE BILINEAR TRANSFORMATION:

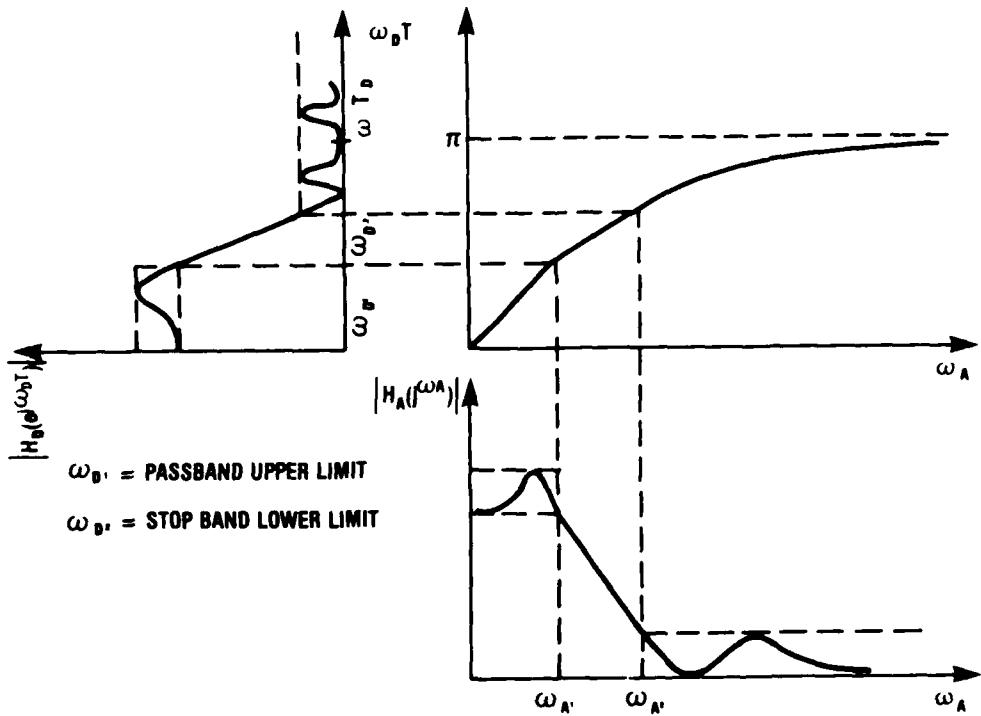


Figure 1. Bilinear transformation.

If only the amplitude response is of concern, the warping effect can for all practical purposes be eliminated by prewarping the analog filter. For example, if a low-pass cutoff frequency of  $\omega_{DC}$  is desired for the digital filter, then the analog filter is first designed for an unnormalized cutoff frequency given by

$$\omega_{AC} = \frac{2}{T} \tan \left( \frac{\omega_{DC}^T}{2} \right).$$

The phase response of the derived digital filter is nonlinear because of the warping effect. Furthermore, little can be done to linearize it except by employing delay equalization. Consequently, if it is mandatory to preserve a linear phase response, alternative methods should be considered.

The bilinear transformation is the most important of the techniques used to obtain digital recursive filters from analog filters. The passband and loss characteristics of the analog filter are preserved, and there is no aliasing effect. Frequency distortion is compensated at  $\omega_{DC}$ , and the transfer function can be obtained by a relatively easy transformation. For these reasons, the bilinear transformation was chosen as the method of obtaining the digital filter transfer function for this project.

#### 2.4 Analog Filter Designs

The theory of analog filter approximations has been extensively developed.<sup>1-4</sup>

##### 2.4.1 Butterworth

The Butterworth approximation is the simplest all-pole type. In the stop band, the attenuation characteristics monotonically decrease as a function of frequency. The Butterworth filter is termed also a maximally flat magnitude approximation in that the error in the passband is also a monotonically decreasing function. The equations for calculating the pole locations are given in appendix A.

---

<sup>1</sup>A. Antoniou, *Digital Filters: Analysis and Design*, McGraw-Hill Book Co., Inc., New York (1979).

<sup>2</sup>M. S. Ghausi, *Principles and Design of Linear Active Circuits*, McGraw-Hill Book Co., Inc., New York (1965), ch. 4.

<sup>3</sup>H. Y.-F. Lam, *Analog and Digital Filters*, Prentice-Hall, Inc., Englewood Cliffs, NJ (1979).

<sup>4</sup>D. E. Johnson, *Introduction to Filter Theory*, Prentice-Hall, Inc., Englewood Cliffs, NJ (1976).

#### 2.4.2 Chebyshev

A second approximation that improves on the rate of change of the attenuation between passband and stop band over that of the Butterworth filter is the Chebyshev filter. The error in the passband is distributed evenly in an oscillating manner. This is called an equiripple approximation. In the stop band, the magnitude decreases monotonically with a faster cutoff rate than that of a Butterworth filter of the same order. The Chebyshev filter is an all-pole type and is based on the nth-order Chebyshev polynomials,  $C_n(\omega)$ .

$$C_n(\omega) = \begin{cases} \cos(n \cos^{-1} \omega), & 0 < \omega < 1, \\ \cosh(n \cosh^{-1} \omega), & \omega > 1. \end{cases}$$

The recursion formula for finding the nth-order polynomial is

$$C_0(\omega) = 1,$$

$$C_1(\omega) = \omega,$$

$$C_n(\omega) = 2\omega C_{n-1}(\omega) - C_{n-2}(\omega).$$

The equations for calculating the pole locations are given in appendix A.

#### 2.4.3 Elliptic

The third type of filter approximation considered in this report is called the elliptic approximation and was first introduced by Wilhelm Cauer. This approximation is based on the Jacobi elliptic sine functions. In this approximation, the error in the passband is again distributed evenly in an oscillating manner. However, instead of a monotonically decreasing characteristic in the stop band, the stop-band attenuation oscillates between infinity and a prescribed maximum. Thus, there is an equiripple characteristic in both passband and stop band. The elliptic approximation is more efficient than the Butterworth and Chebyshev approximations in that the transition between passband and stop band is steeper for a given filter order. An elliptic filter transfer function has both poles and zeros and has been shown to be optimal in the sense of having the sharpest transition of any approximation. The technique used to calculate the coefficients for the elliptic filter is given in appendix A and was taken from the development by Antoniou.<sup>1</sup>

---

<sup>1</sup>A. Antoniou, *Digital Filters: Analysis and Design*, McGraw-Hill Book Co., Inc., New York (1979).

## 2.5 Digital Filter Realization

There are two possible canonical forms of linear difference equations that can be realized from the  $z$  domain transfer function,  $H(z)$ . One, the cascade form, follows from the factored form of  $H(z)$ . The other, the parallel form, requires the expansion of  $H(z)$  into partial fractions. The parallel form was chosen for this project because of its reduced sensitivity to noise.

Realization of a recursive digital filter in parallel form is shown in figure 2.

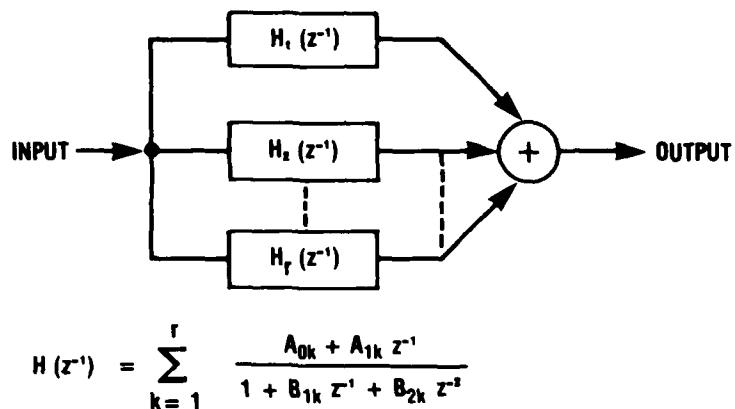


Figure 2. Recursive digital filter in parallel form.

## 2.6 Band Transformations

For this project, high-pass, band-pass, and band-stop filters are obtained from the digital low-pass prototype by a transformation in the  $z$  domain. Another approach is to design the analog low-pass prototype, perform the transformation in the  $s$  domain to high pass, band pass, or band stop, and then transform the resulting transfer function to the  $z$  domain by the bilinear transformation.

The work of Constantinides<sup>5</sup> was used to develop the  $z$  domain transformations. The equations are given in appendix A.

---

<sup>5</sup>A. G. Constantinides, *Spectral Transformations for Digital Filters*, Proc. IEE, 117 (August 1970), 1585-1590.

### 3. FORTRAN IV PROGRAM

#### 3.1 Outline

A FORTRAN IV computer program was written using the equations and the techniques discussed in section 2. The program was designed in a modular form to allow for easy modification as required. The flow chart of the program is given in figure 3. The main part of the program is simply a controller that calls in subroutines as required. The blocks in a column in the center of the flow chart are all in the main program, and the blocks to the right or the left of center are separate subroutines. For instance, if a different form of input data is required or a different output format is desired, then only the subroutine dealing specifically with that function need be modified. If a different analog design type or technique is to be used, it can be either substituted for one of the existing three design types or added on by extending a "computed go to" statement in the main program.

All data are stored in labeled and unlabeled common areas of a subroutine simply by including the proper common statement.

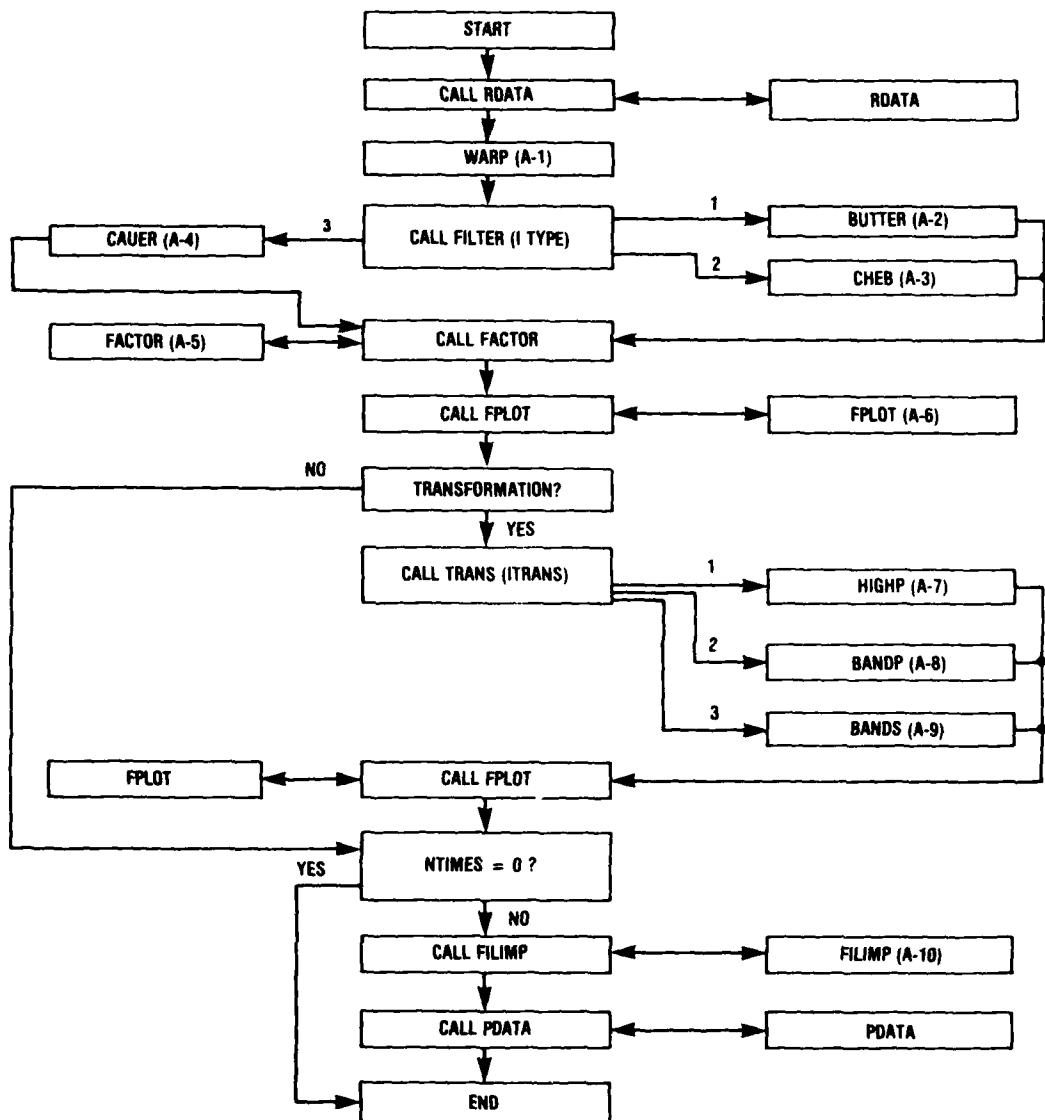
This program was developed by using an IBM System/370 computer. The H Extended Plus FORTRAN compiler was used with the auto-double option specified. This option doubles the overall precision specified in the program. Maximum possible precision is used for all the design calculations as well as in the implementation itself. This precision is 16 bytes for real variables and 32 bytes for complex variables. The complete program listing is given in appendix B.

The flow of the program is straightforward because it begins at the top and progresses without any diversions to the bottom of the chart. The basic steps of the program and the general function of each subroutine are as follows.

##### 3.1.1 RDATA

After the program is initiated, it immediately calls the RDATA subroutine. This subroutine reads in the necessary filter design criteria and the data samples to be processed. Only batch processing of data is possible with the existing program, but the conversion to real time processing would not be difficult. The input data array is dimensioned for up to 1000 amplitude points at the specified sampling frequency. If only a filter design is required, then the number of data points (NTIMES) is set equal to zero, and RDATA bypasses the input of data to be processed. Setting NTIMES equal to zero bypasses also the implementation subroutine. RDATA also prints out the design criteria data for future reference.

The specific format required for data input is given in section 3.2.



NOTE: NUMBERS IN PARENTHESES CORRESPOND TO SECTIONS IN APPENDIX A.

Figure 3. Recursive digital filter program.

### 3.1.2 WARP

The RDATA subroutine returns control to the main program, which calculates the prewarped analog design frequency (WARP) and then calls the appropriate analog design subroutine. The analog filter subroutines calculate the poles and the zeros (if any) of the squared magnitude transfer function that lie in the left half of the complex s plane. The resulting transfer function after the right-half plane poles and zeros are eliminated is of the form

$$H(s) = \frac{(AMP)(s - z_1)(s - z_2) \dots (s - \bar{z}_1)}{(s - p_1)(s - p_2) \dots (s - \bar{p}_1)},$$

where  $\bar{z}_i$  and  $\bar{p}_i$  are complex conjugates of  $z_i$  and  $p_i$  (for all  $i$ ). The equations needed to calculate these poles and zeros are given in appendix A and are derived as discussed in section 2.

### 3.1.3 FACTOR

The FACTOR subroutine is then called. FACTOR expands the transfer function into partial fractions. If the numbers of poles and zeros are equal, then this expansion has a constant gain equal to the amplitude multiplier (AMP) of the transfer function. FACTOR then combines the complex conjugate pairs and applies the bilinear transformation,

$$s = \frac{2}{T} \frac{1 - z^{-1}}{1 + z^{-1}},$$

to obtain the coefficients for the parallel second-order sections of the digital filter. The general form of the resulting z domain transfer function is<sup>6</sup>

$$H(z^{-1}) = (1 + z^{-1}) \sum_{i=1}^r \frac{A_{0i} + A_{1i}z^{-1}}{1 + B_{1i}z^{-1} + B_{2i}z^{-2}},$$

where  $r = n/2$  for  $n$  even,  $r = (n + 1)/2$  for  $n$  odd, and  $n$  is the order of the filter. The coefficients  $A_{0i}$ ,  $A_{1i}$ ,  $B_{1i}$ , and  $B_{2i}$  are calculated by FACTOR using the analog poles and zeros. If the order of the filter is odd, then there is one real pole, so the  $A_1$  and  $B_2$  coefficients are equal to zero for that pole.

<sup>6</sup>G. C. Temes and S. K. Mitra, *Modern Filter Theory and Design*, John Wiley & Sons, Inc., New York (1973).

### 3.1.4 F PLOT

A plot of the frequency response (magnitude and phase) of the low-pass digital filter is then made by the F PLOT subroutine. It is plotted by letting

$$z^{-1} = e^{-j\omega T} = \cos \omega T - j \sin \omega T$$

in the z domain transfer function given above.

The plots given in this report were made on a Houston Instruments plotter using existing plotting software.<sup>7</sup> Both the amplitude and phase responses are plotted over the normalized range of frequencies of zero to one. That is, the frequency scale is normalized to the sampling frequency. F PLOT also prints the values of the digital filter coefficients to 25 decimal places for future use.

### 3.1.5 Filter Choice

The main program then determines if a high-pass, band-pass, or band-stop filter is desired or if the low-pass filter is ready to be implemented.

The transfer function for the required high-pass, band-pass, or band-stop filter is obtained by an appropriate transformation in the z domain of the low-pass digital filter. Band-pass and band-stop transformations result in a doubling of the filter order.

After transformation, the frequency characteristics of the new filter are plotted by F PLOT.

### 3.1.6 FILIMP

The actual implementation of the digital filter is simple. The FILIMP subroutine performs this function and processes the data to be filtered. Application of the inverse z transform to the general form for the z domain transfer function results in the recursive difference equation

$$Y_{out}(kT) = X(kT) + X[(k - 1)T] ,$$

---

<sup>7</sup>Thomas V. Noon and Egon Marx, *User's Manual for the Modular Analysis-Package Libraries ANAPAC and TRANL, Harry Diamond Laboratories HDL-TR-1782-S (September 1978)*.

where

$$X(kT) = \sum_{i=1}^r X_i(kT) ,$$

$$X_i(kT) = A_{0i}Y_{in}(kT) + A_{1i}Y_{in}[(k-1)T] \\ - B_{1i}X_i[(k-1)T] - B_{2i}X_i[(k-2)T] .$$

### 3.1.7 PDATA

The PDATA subroutine provides plots of the input and output data. This routine may be easily modified to provide the output data in any desired format.

### 3.2 Program Use

There are at most nine parameters that must be supplied to the program for the design of a filter. The minimum number needed is five for the design of a Butterworth low-pass filter. This minimum set, which is used in all designs, consists of these parameters:

- a. ITYPE, the type of analog design

1 = Butterworth

2 = Chebyshev

3 = Elliptic (Cauer)

- b. N, the order of the filter

- c. ITRANS, corresponding to the band transformation desired

0 = None

1 = High-pass transformation

2 = Band-pass transformation

3 = Band-stop transformation

- d. FC, the cutoff frequency desired (in hertz)

- e. FS, the sampling frequency of the data (in hertz) to which the filter will be normalized

If a low-pass Chebyshev is required, then an additional parameter, EPS1, the minimum allowed amplitude in the passband, also must be entered. Similarly, for elliptic filters, EPS1 must be given along with EPS2, the transition region selectivity factor. The maximum value for EPS2 is 0.95. All of these parameters are read from the same data card. These are the formats for all the data cards and the order in which the cards are read by the program:

ITYPE,N,ITRANS,FC,FS,EPS1,EPS2  
Format--3I5, 4E10.3

If ITRANS = 0, skip reading F1 and F2.

F1,F2  
Format--2E10.3

NTIMES  
Format--I5

If NTIMES = 0, skip reading VIN.

VIN  
Format--8E10.3--maximum of 125 cards

Heading for low-pass design

If ITRANS = 0, skip next heading.

Heading for band transformation

If NTIMES = 0, skip next two headings.

Heading for VIN plot

Heading for VOUT plot

If a band transformation is specified, then an additional data card is needed that has the lower cutoff frequency, F1 (also the high-pass cutoff frequency), and the upper cutoff frequency, F2, for the band-pass and band-stop filters.

Next, the number of data sample points is read. If NTIMES is set equal to zero, then no input data are read, and the program is terminated after completion of the filter design.

Separate titles are read in for the low-pass filter transfer function plots, any band transformation frequency plots, and the heading to appear on the data plots. These titles can be up to 80 characters of any form desired.

A sample filter design is given now to illustrate the order and the manner in which the data are assembled. A fourth-order elliptic band-pass filter is chosen since this design requires input of all the design parameters. We have this:

ITYPE = 3

N = 4

ITRANS = 2

Now let us assume that the sampling frequency is 10,000 Hz and that the passband is to be from 1000 to 3000 Hz. In addition, we allow the passband amplitude to vary between 1.0 and 0.9 and minimize the transition between the passband and the stop band. This action means that the cutoff frequency for the low-pass design must be 2000 Hz, which is the width of the passband. Therefore, the remaining parameters are these:

FC = 2000.0

FS = 10000.0

EPS1 = 0.9

EPS2 = 0.95

F1 = 1000.0

F2 = 3000.0

These data are given in figure 4 as they appear on the input data cards. The plot title cards also are shown in the figure. If only a low-pass design is requested, then the second title card is omitted. Similarly, if no data points are given, then the third title card as well as the input data cards are omitted.

The resulting printout for this example problem is given in figure 5. The low-pass transfer function is given in figures 6 and 7, and the band-pass function is given in figures 8 and 9. Running this example required 6.85 s in the central processing unit (CPU) to compile the FORTRAN coding, 2.44 s in the CPU to link and edit the object modules with library functions, and about 2 s in the CPU to perform the actual calculations. The filtering of 101 data points required less than 2 s of CPU time. Additional examples of filter designs are given in section 4.

Figure 4. Input data for example 1.

Figure 5. Output data for example 1.

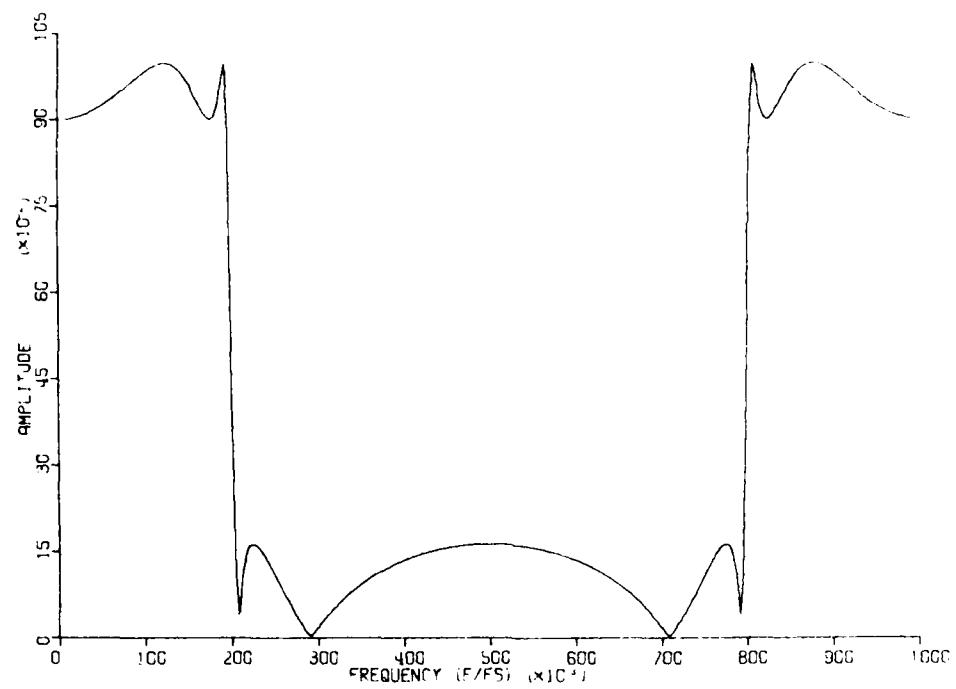


Figure 6. Low-pass Cauer filter: amplitude versus frequency.

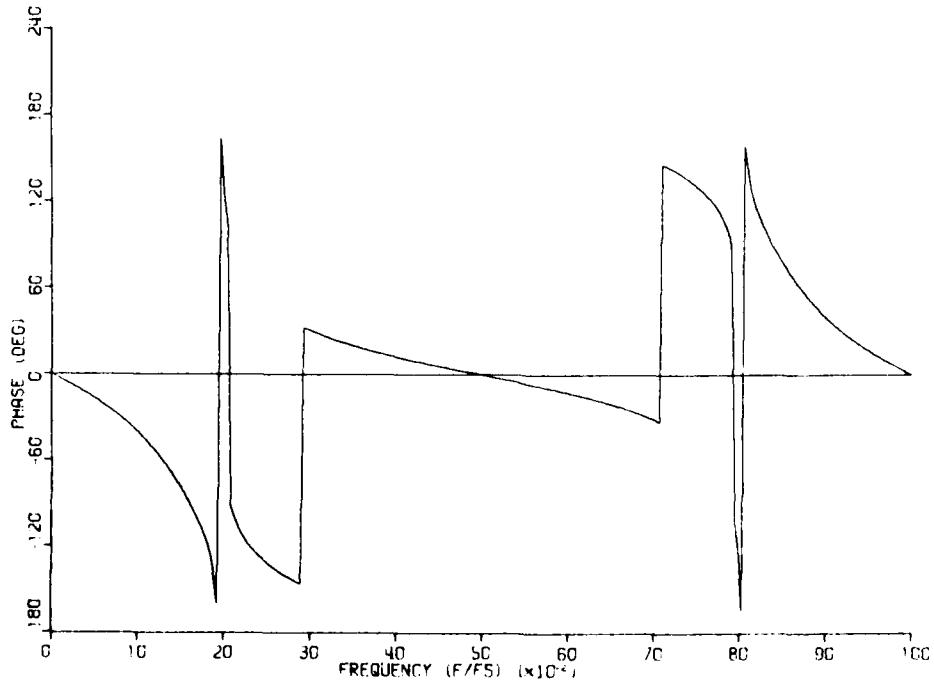


Figure 7. Low-pass Cauer filter: phase versus frequency.

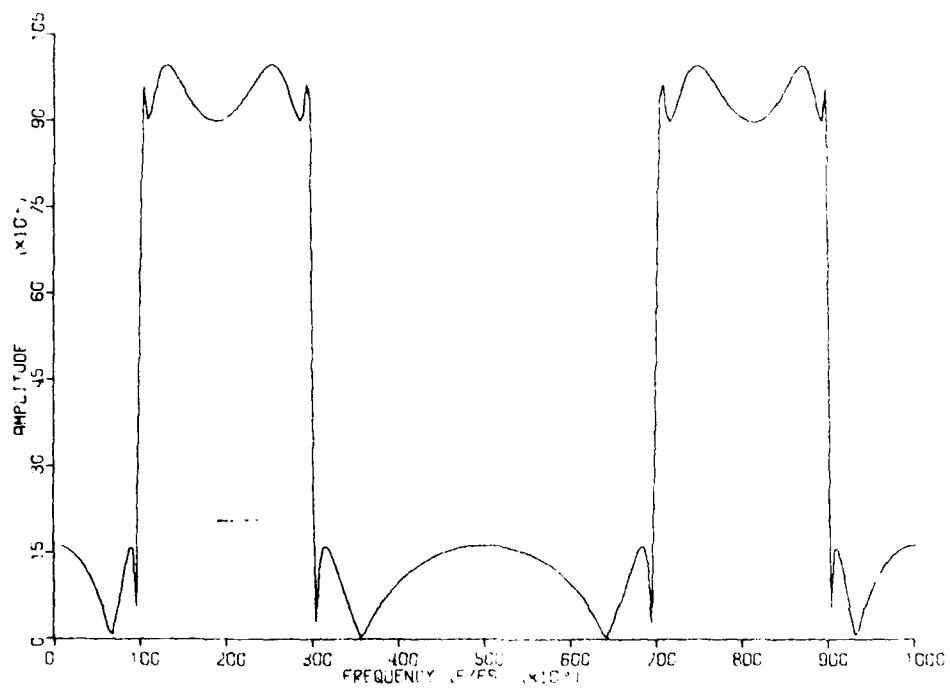


Figure 8. Band-pass Cauer filter: amplitude versus frequency.

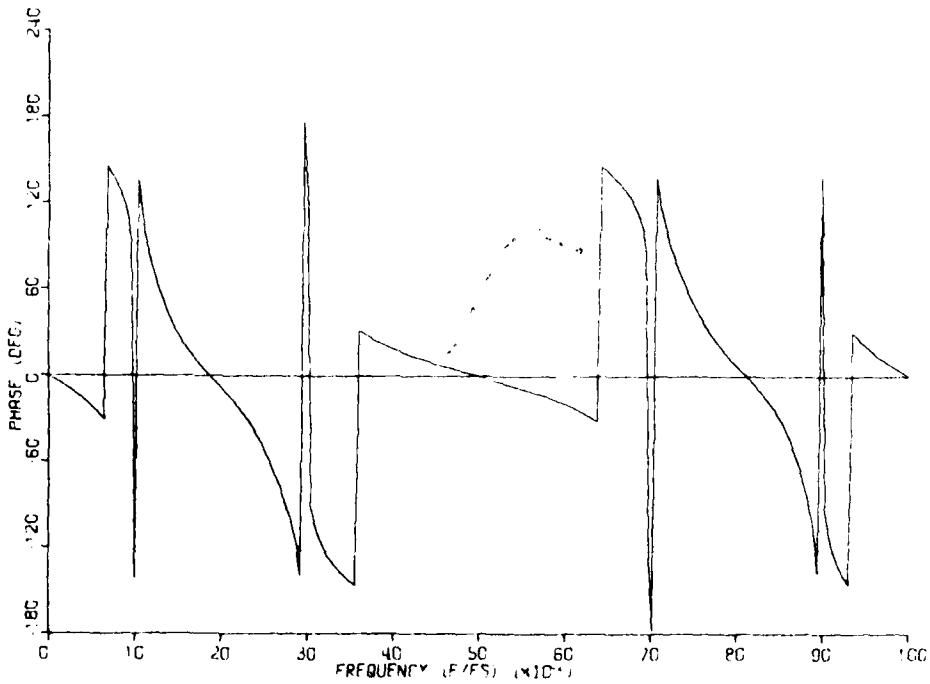


Figure 9. Band-pass Cauer filter: phase versus frequency.

#### 4. DESIGN EXAMPLES

Several examples are given in this section to illustrate the differences in the various analog designs and also to show the results of band transformations. In addition, a simple waveform consisting of three sinusoids at different frequencies is used to demonstrate the filtering action of the various digital filters.

##### 4.1 Analog Design

The three types of analog designs are discussed in section 2. A graphical presentation of the basic differences in these three designs is given here. A fourth-order elliptic low-pass design is given in section 3.2. Similar fourth-order digital filters using the Butterworth and Chebyshev analog designs are presented in figures 10 to 13. Comparing these transfer functions, we can easily see that the Butterworth filter offers the smoothest response in both the passband and the stop band. The sharper transition from passband to stop band of the Chebyshev design over the Butterworth is obtained at the expense of smoothness in the passband. The elliptic design obviously has the sharpest transition of the three designs. However, the ripple in both the passband and the stop band is the penalty for this sharp transition. The design to use is chosen according to the requirements of the specific filtering problem being considered.

##### 4.2 Band Transformations

An example of a band-pass transformation is given in figures 8 and 9. The low-pass Butterworth filter of figures 10 and 11 was transformed to a high-pass filter with a cutoff of 3000 Hz as shown in figures 14 and 15. The band-stop dual of the passband filter was made by using the Chebyshev design and is shown in figures 16 and 17.

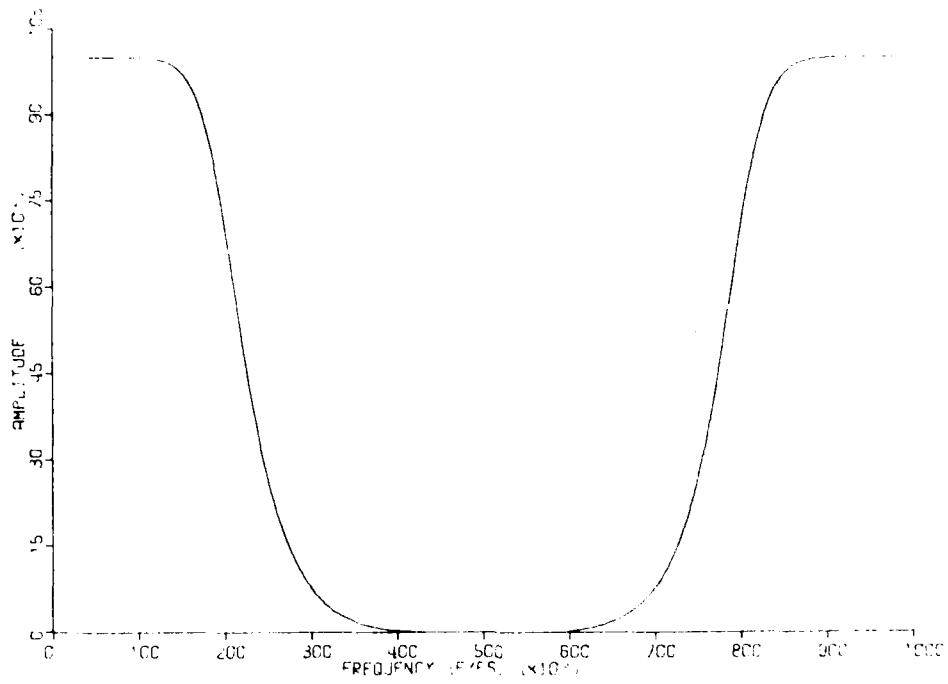


Figure 10. Low-pass Butterworth filter: amplitude versus frequency.

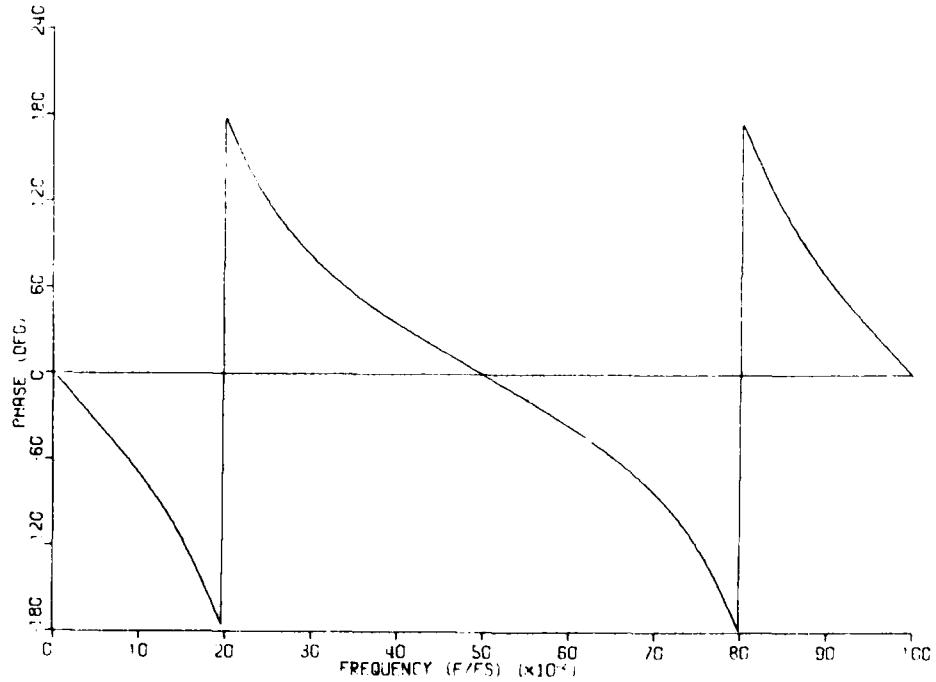


Figure 11. Low-pass Butterworth filter: phase versus frequency.

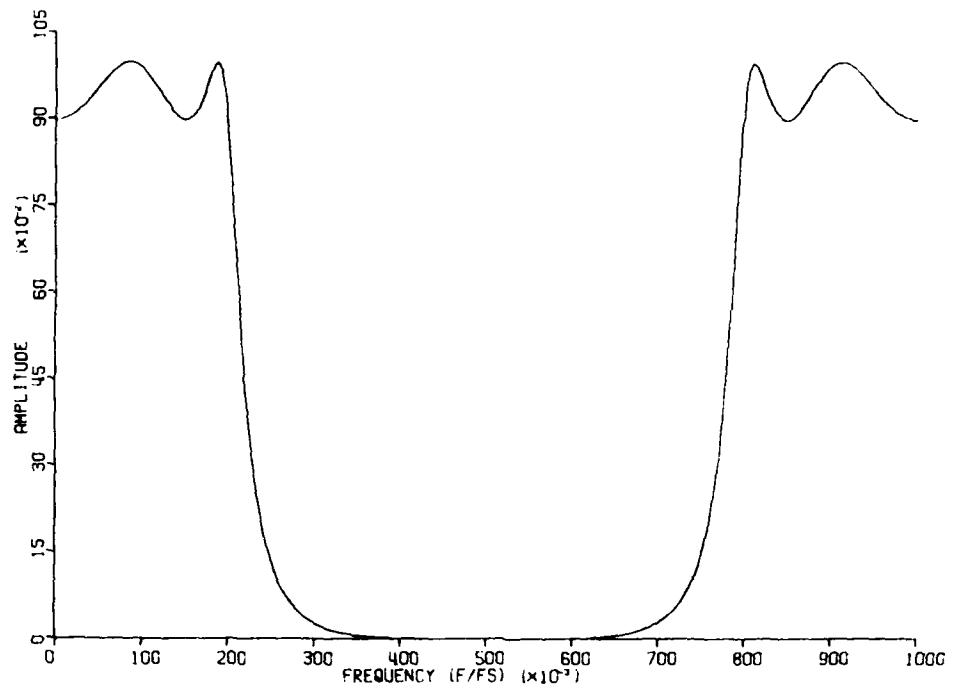


Figure 12. Low-pass Chebyshev filter: amplitude versus frequency.

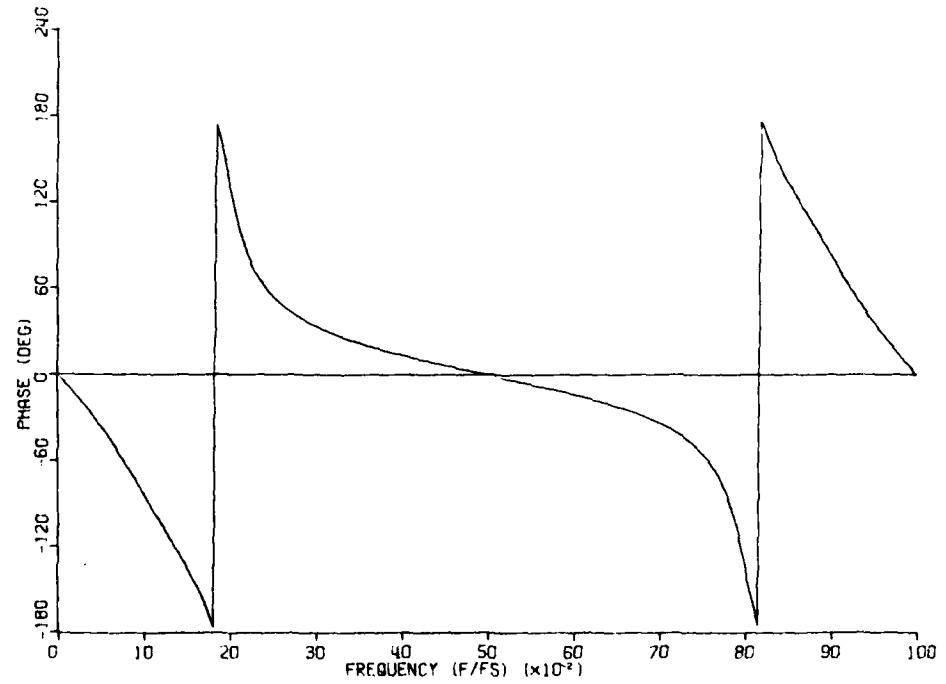


Figure 13. Low-pass Chebyshev filter: phase versus frequency.

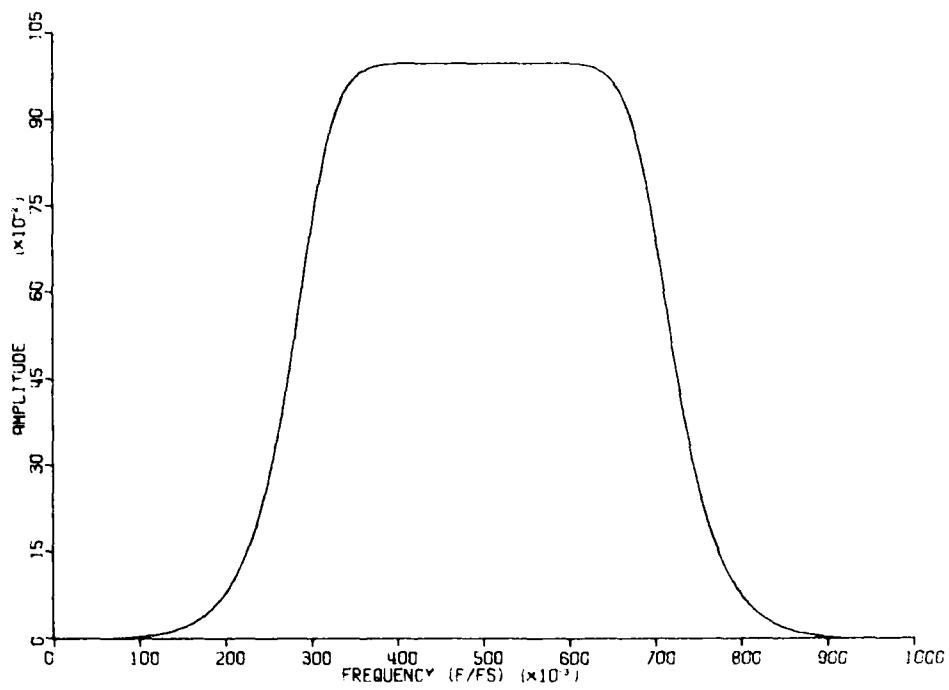


Figure 14. High-pass Butterworth filter: amplitude versus frequency

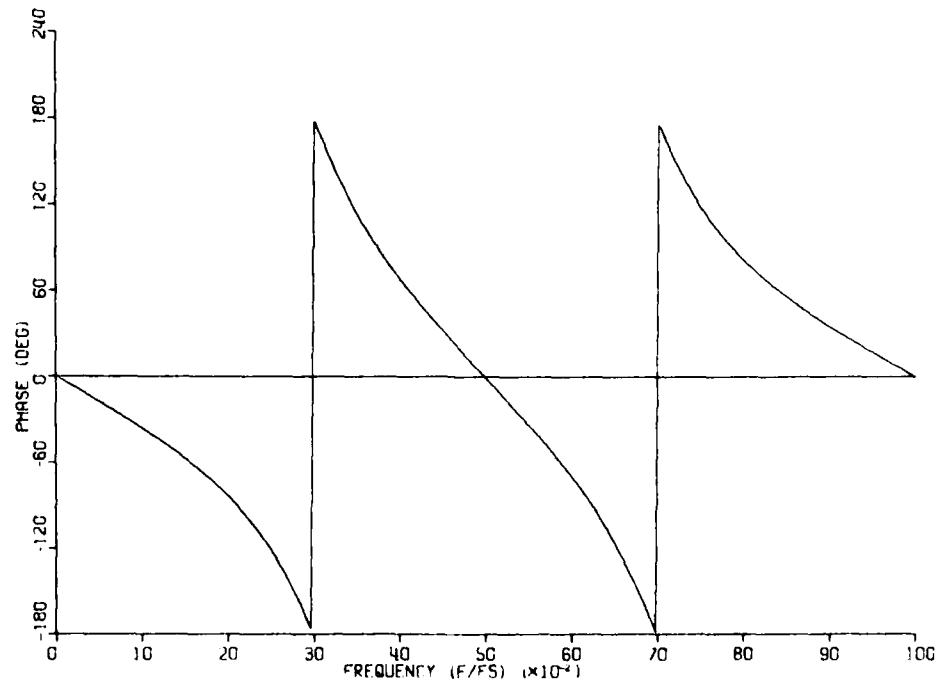


Figure 15. High-pass Butterworth filter: phase versus frequency.

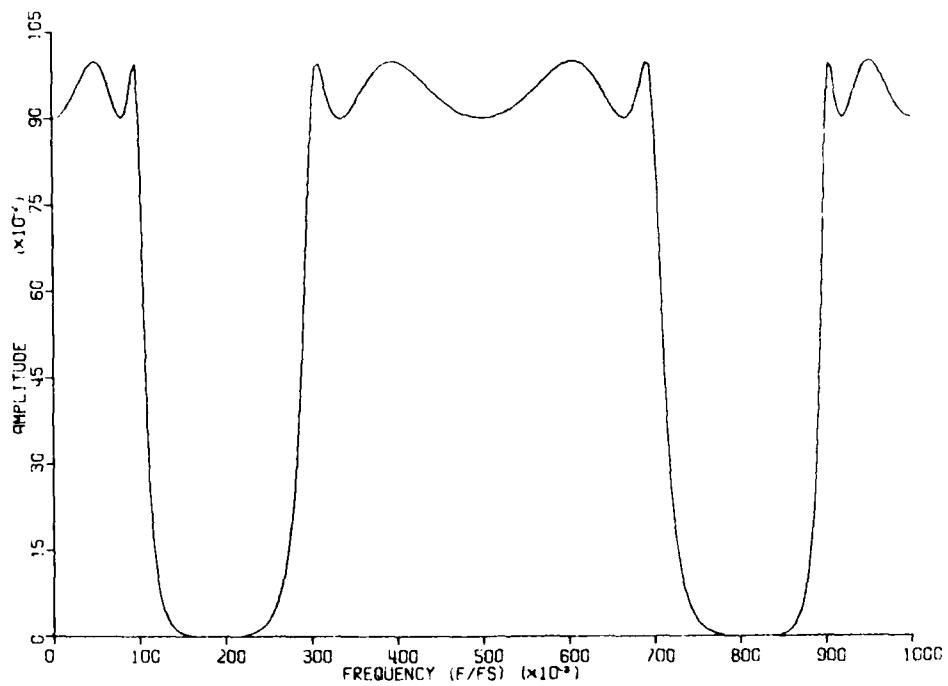


Figure 16. Band-stop Chebyshev filter: amplitude versus frequency.

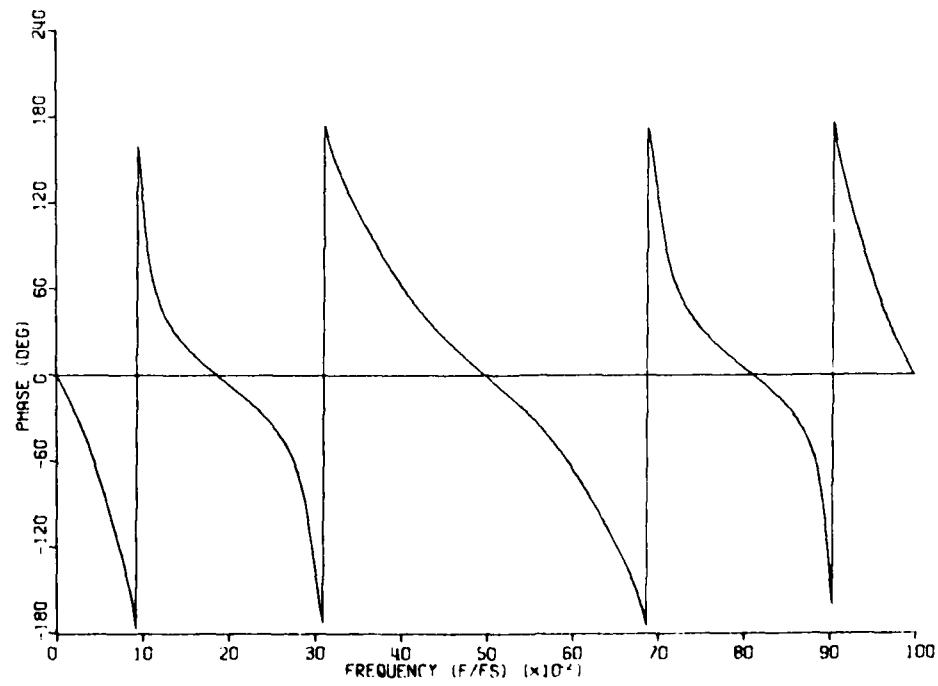


Figure 17. Band-stop Chebyshev filter: phase versus frequency.

#### 4.3 Data Processing

A simple waveform was generated to illustrate the operation of the filters on input data. The waveform used was the sum of three sinusoids at 0.05, 0.15, and 0.35 times the sampling frequency with peak amplitudes of 3, 2, and 5, respectively. This input waveform is shown in figure 18. A 20th-order Butterworth low-pass design was used to eliminate the two highest frequency sine waves, as shown in figure 19. A filter of high order was used to demonstrate the group delay effect of the filter. This effect shows at the beginning of the output waveform as a delay time before any output occurs. Next, a fourth-order Butterworth high-pass filter was used to eliminate the two low-frequency signals. This result is shown in figure 20. The low sampling rate caused the distortion on the sine wave seen in this figure. The initial design example was used to pass only the center sinusoid as shown in figure 21. Here, again, the effect of the low sampling rate is seen as a distortion of the waveform. However, the effect is not as great because of the lower frequency of the sinusoid. The Chebyshev band-stop dual of this passband filter was then used to eliminate the middle sine wave. This waveform is given in figure 22.

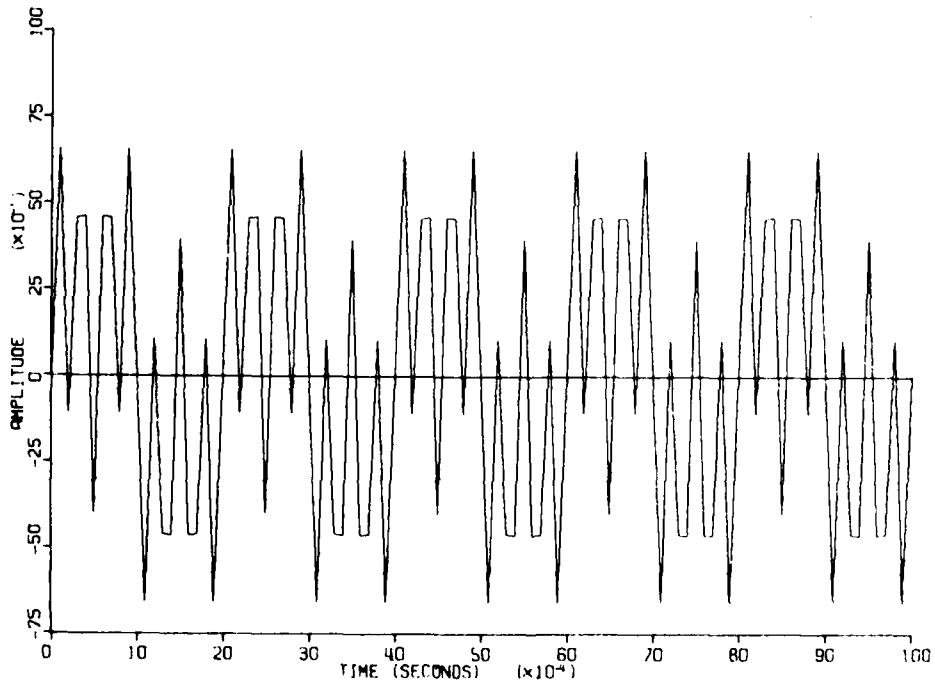


Figure 18. Input waveform obtained by summing three sinusoids.

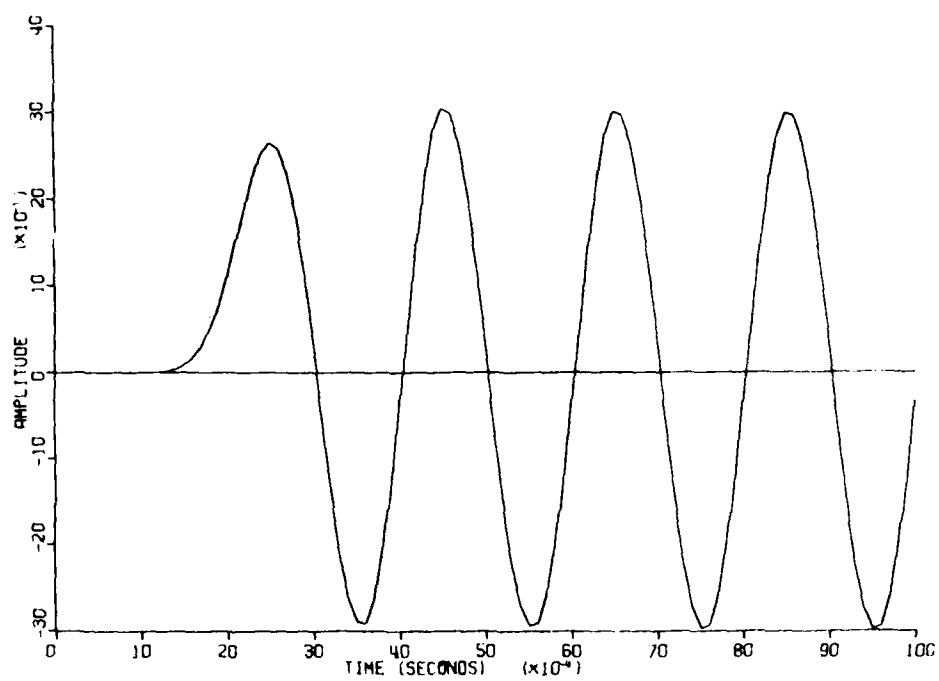


Figure 19. Output waveform obtained by using low-pass Butterworth filter.

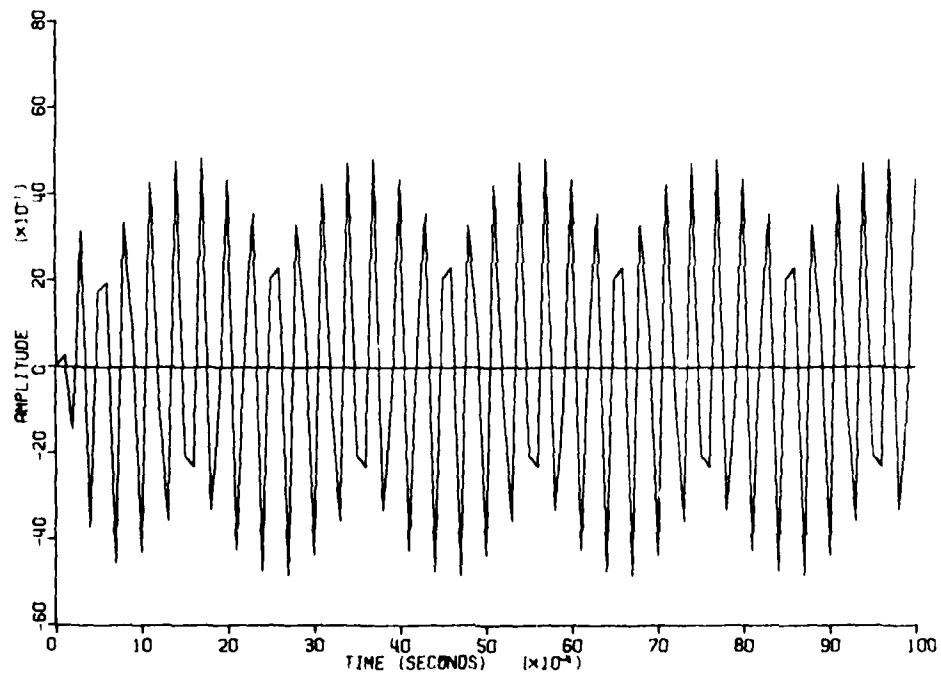


Figure 20. Output waveform obtained by using high-pass Butterworth filter.

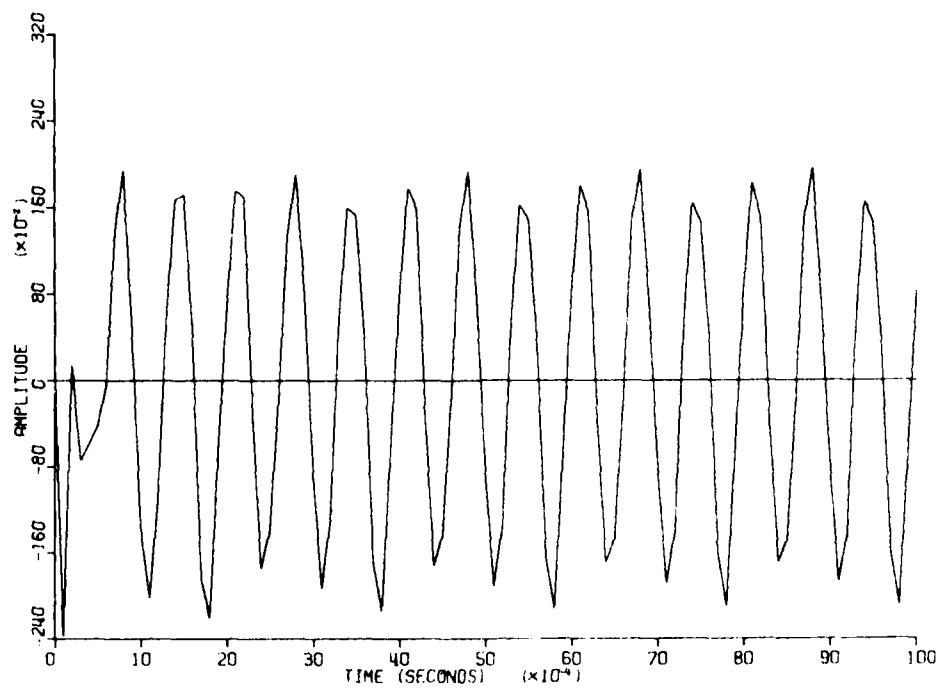


Figure 21. Output waveform obtained by using band-pass Cauer filter.

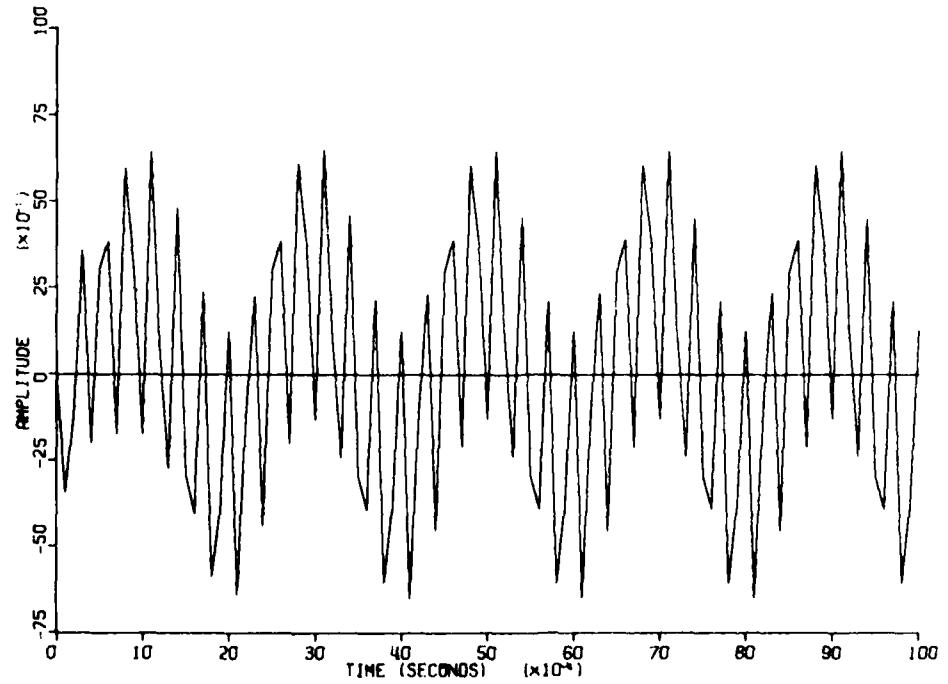


Figure 22. Output waveform obtained by using band-stop Chebyshev filter.

## 5. CONCLUSIONS AND RECOMMENDATIONS

A versatile program has been developed for designing and implementing recursive digital filters. Three well-known analog filter designs were used with the bilinear transformation to obtain the desired digital filters. The bilinear technique was chosen because it eliminates the effects of aliasing that occur with other techniques. However, the resulting warping of the frequency scale may not be acceptable in some applications. The extent to which this warping affects the resulting output was not investigated. Examples using the different analog designs are given in section 4 along with transformations from low-pass to high-pass, band-pass, and band-stop designs. All of the filter design calculations as well as the implementations use the highest precision possible for the IBM System/370 computer. No investigation was made to determine the effects of using lower precision.

All of the equations used in developing the code are included in appendix A, and the complete computer program code is listed in appendix B. A bibliography of related publications also is included. This bibliography is by no means complete since the amount of information published seems almost endless. Although there is a large amount of information available, no one source proved adequate in presenting all of the steps necessary to completely develop a digital filter. It is hoped, therefore, that this document will prove helpful to others venturing into this field for the first time.

Although the program developed here is completely operable, there are many ways in which the program may be improved or modified to better suit a particular filtering need. Also, investigation and analysis of the various errors associated with the techniques used should be conducted.

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APPENDIX A.--EQUATIONS USED FOR CODE DEVELOPMENT

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## APPENDIX A

The following equations were used in the development of the computer code for designing and implementing digital filters. All of the equations are referenced in the main body of this report. The application of each set of equations is given in the flow chart of the program (fig. 3).

### A-1. PREWARPING (WARP)

$$\tilde{f}_c = \frac{f_{AC}}{f_s} = \frac{1}{\pi} \tan \left( \pi \frac{f_{DC}}{f_s} \right).$$

### A-2. BUTTERWORTH

The transfer function for a Butterworth low-pass filter of order  $n$  is given by

$$H(s) = \frac{H_0}{\prod_{k=1}^n (s - p_k)},$$

where

$$H_0 = \tilde{\omega}_c^n,$$

$\tilde{\omega}_c$  = cutoff frequency,

$$p_k = \tilde{\omega}_c e^{j[(\pi/2) + \pi(2k-1)/2n]}, \quad k = 1, 2, \dots, n.$$

### A-3. CHEBYSHEV

The squared magnitude function is of the form

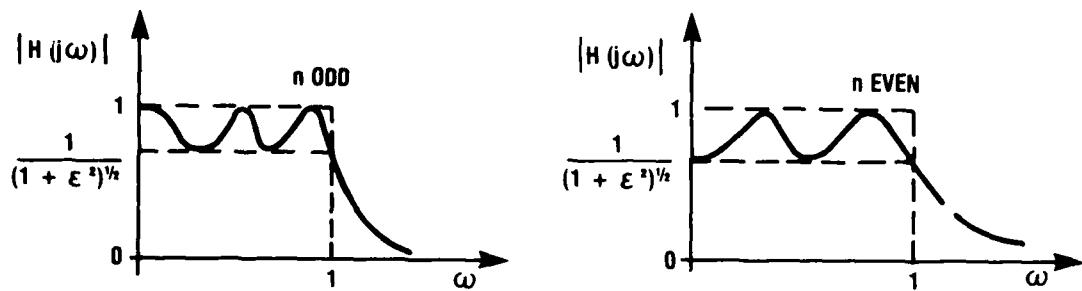
$$|H(j\omega)|^2 = \frac{1}{1 + \epsilon^2 C_n^2(\omega)},$$

where

$\epsilon$  = ripple factor,

$C_n(\omega)$  = Chebyshev polynomial.

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For the program,  $\epsilon$  is entered as

$$\text{EPS1} = 1 - \frac{1}{(1 + \epsilon^2)^{1/2}} .$$

The transfer function is of the form

$$H(s) = \frac{H_0}{\prod_{k=1}^n (s - p_k)} ,$$

where

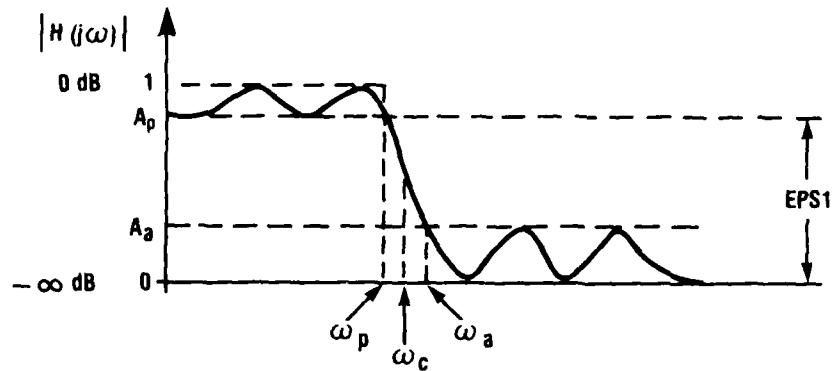
$$p_k = \sin(u_k) \sinh(v) + j \cos(u_k) \cosh(v) = \alpha_k + j\beta_k ,$$

$$u_k = \frac{\pi}{2n} (2k - 1), \quad k = n + 1, n + 2, \dots, 2n ,$$

$$v = \frac{1}{n} \sinh^{-1} \left( \frac{1}{\epsilon} \right) ,$$

$$H_0 = \begin{cases} \left[ \alpha_{(n+1)/2} \right] \prod_{k=1}^{(n-1)/2} (\alpha_k^2 + \beta_k^2), & n \text{ odd,} \\ \frac{1}{(1 + \epsilon^2)^{1/2}} \prod_{k=1}^{n/2} (\alpha_k^2 + \beta_k^2), & n \text{ even.} \end{cases}$$

A-4. ELLIPTIC (CAUER)



For the normalized filter  $\omega_c = 1$ ,

$n$  = filter order,

$$\omega_c = (\omega_p \omega_a)^{1/2} = 1,$$

$k$  = selectivity factor  $= \omega_p / \omega_a$ ,

$A_p$  = maximum passband attenuation (in decibels),

$A_a$  = minimum stop-band attenuation (in decibels).

Of the parameters  $A_p$ ,  $A_a$ ,  $k$ , and  $n$ , only three are independent. If three of the four are specified, the fourth is automatically fixed. For this project,  $n$ ,  $k$ , and  $A_p$  are specified:

$k = \text{EPS2}$ ,

$$A_p = 20 \log (1 - \text{EPS1}).$$

$A_a$  is then given by

$$A_a = 10 \log \left( \frac{10^{-0.1A_p} - 1}{16q^n} + 1 \right).$$

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The normalized elliptic low-pass filter has a transfer function of the form

$$H_N(s) = \frac{H_0}{D_0(s)} \prod_{i=1}^r \frac{s^2 + A_{0i}}{s^2 + B_{1i}s + B_{0i}},$$

where

$$r = \begin{cases} (n-1)/2, & n \text{ odd}, \\ n/2, & n \text{ even}, \end{cases}$$

$$D_0(s) = \begin{cases} s + \sigma_0, & n \text{ odd}, \\ 1, & n \text{ even}. \end{cases}$$

The transfer function coefficients and multiplier constant  $H_0$  are computed by using the following formulas in sequence:

$$k' = (1 - k^2)^{1/2}, \quad (A-1)$$

$$q_0 = \frac{1}{2} \frac{1 - \sqrt{k'}}{1 + \sqrt{k'}}, \quad (A-2)$$

$$q = q_0 + 2q_0^5 + 15q_0^9 + 150q_0^{13}, \quad (A-3)$$

$$\Lambda = \frac{1}{2n} \ln \left( \frac{\frac{10}{-0.05A_p} + 1}{\frac{10}{-0.05A_p} - 1} \right), \quad (A-4)$$

$$\sigma_0 = \left| \frac{2q^{1/4} \sum_{m=0}^{\infty} (-1)^m q^{m(m+1)} \sinh [(2m+1)\Lambda]}{1 + 2 \sum_{m=1}^{\infty} (-1)^m q^{m^2} \cosh (2m\Lambda)} \right|, \quad (A-5)$$

$$\Omega_i = \frac{2q^{1/4} \sum_{m=0}^{\infty} (-1)^m q^{m(m+1)} \sin \left[ \frac{(2m+1)\pi u}{n} \right]}{1 + 2 \sum_{m=1}^{\infty} (-1)^m q^{m^2} \cos \left( \frac{2m\pi u}{n} \right)}, \quad (A-6)$$

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where

$$\mu = \begin{cases} i, & n \text{ odd}, \\ i - \frac{1}{2}, & n \text{ even}, \end{cases} \quad i = 1, 2, \dots, r.$$

The series in equations (A-5) and (A-6) converge rapidly, and usually three or four terms are sufficient.

$$w = \left[ (1 + k\sigma_0^2) (1 + \sigma_0^2/k) \right]^{1/2}, \quad (A-7)$$

$$v_i = \left[ (1 - k\Omega_i^2) (1 - \Omega_i^2/k) \right]^{1/2}, \quad (A-8)$$

$$A_{0i} = 1/\Omega_i^2, \quad (A-9)$$

$$B_{0i} = \frac{(\sigma_0 v_i)^2 + (\Omega_i w)^2}{(1 + \sigma_0^2 \Omega_i^2)^2}, \quad (A-10)$$

$$B_{1i} = \frac{2\sigma_0 v_i}{1 + \sigma_0^2 \Omega_i^2}, \quad (A-11)$$

$$H_0 = \begin{cases} \sigma_0 \prod_{i=1}^r \frac{B_{0i}}{A_{0i}}, & n \text{ odd}, \\ 10^{-0.05A_p} \prod_{i=1}^r \frac{B_{0i}}{A_{0i}}, & n \text{ even}. \end{cases} \quad (A-12)$$

A-5. FACTOR

$$R_k + jI_k = \frac{H_0 \prod_{m=1}^{n'} (p_k - z_m)}{\prod_{\ell=1}^n (p_k - p_\ell)}, \quad \ell \neq k, \quad n' = \begin{cases} n, & n \text{ even}, \\ n-1, & n \text{ odd}, \end{cases}$$

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$$H(s) = \sum_{k=1}^r \left( \frac{R_k + jI_k}{s - p_k} + \frac{R_k - jI_k}{s - \bar{p}_k} \right), \quad r = \begin{cases} (n+1)/2, & n \text{ odd}, \\ n/2, & n \text{ even}, \end{cases}$$

$$s = \frac{2}{T} \frac{1 - z^{-1}}{1 + z^{-1}},$$

$$H(z^{-1}) = (1 + z^{-1}) \sum_{k=1}^r \frac{A_{0k} + A_{1k}z^{-1}}{1 + B_{1k}z^{-1} + B_{2k}z^{-2}},$$

$$p_k = -\alpha_k + j\beta_k,$$

$$A_{0k} = [R_k(1 + \alpha_k/2) - I_k(\beta_k/2)]/D,$$

$$A_{1k} = -[R_k(1 - \alpha_k/2) + I_k(\beta_k/2)]/D,$$

$$B_{1k} = -2[1 - (\alpha_k/2)^2 - (\beta_k/2)^2]/D,$$

$$B_{2k} = [(\alpha_k/2)^2 + (\beta_k/2)^2]/D,$$

$$D = (1 + \alpha_k/2)^2 + (\beta_k/2)^2.$$

### A-6. MAGNITUDE AND PHASE (FPLOT)

The frequency response of the designed digital filter is calculated by substituting  $e^{-j\omega T}$  for  $z^{-1}$  in the transfer function and then using complex arithmetic to obtain the magnitude and the phase as a function of frequency.

$$z^{-1} + e^{-j\omega T} = w = \cos \omega T - j \sin \omega T,$$

$$H(w) = (1 + w) \sum_{k=1}^r \frac{A_{0k} + A_{1k}w}{1 + B_{1k}w + B_{2k}w^2}.$$

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## A-7. HIGH-PASS TRANSFORMATION

A low-pass digital filter of cutoff frequency  $\theta_p$  is first designed for the desired order,  $n$ , and type. Let  $\omega_p$  = the desired high-pass cutoff frequency.

Then substituting for  $z^{-1}$  in the low-pass transfer function,

$$z^{-1} \rightarrow -\frac{z^{-1} + \alpha}{1 + \alpha z^{-1}} ,$$

where

$$\alpha = -\frac{\cos\left(\frac{\omega_p + \theta_p}{2}\right)}{\cos\left(\frac{\omega_p - \theta_p}{2}\right)} ,$$

results in the transfer function of the desired high-pass filter.

$$H_{HP}(z^{-1}) = (1 - z^{-1}) \sum_{k=1}^r \frac{A'_{0k} + A'_{1k}z^{-1}}{1 + B'_{1k}z^{-1} + B'_{2k}z^{-2}} ,$$

$$A'_{0k} = (1 - \alpha) \left( \frac{A_{0k} - \alpha A_{1k}}{D'} \right) ,$$

$$A'_{1k} = (1 - \alpha) \left( \frac{\alpha A_{0k} - A_{1k}}{D'} \right) ,$$

$$B'_{1k} = [2\alpha - B_{1k}(1 + \alpha^2) + 2B_{2k}\alpha]/D' ,$$

$$B'_{2k} = (\alpha^2 - \alpha B_{1k} + B_{2k})/D' ,$$

$$D' = 1 - \alpha B_{1k} + \alpha^2 B_{2k} .$$

## A-8. BAND-PASS TRANSFORMATION

A low-pass digital filter of cutoff frequency  $\beta$  is first designed for the desired order and type. The cutoff frequency equals the bandwidth of the desired band-pass filter,  $F_2 - F_1$ , where  $F_2$  and  $F_1$  are the upper and lower cutoff frequencies (in hertz), respectively.

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A substitution for  $z^{-1}$  in the low-pass transfer function results in the transfer function of the desired band-pass filter.

In general, this transformation is<sup>1,2</sup>

$$z^{-1} \rightarrow - \frac{z^{-2} - \frac{2\alpha k}{k+1} z^{-1} + \frac{k-1}{k+1}}{\frac{k-1}{k+1} z^{-2} - \frac{2\alpha k}{k+1} z^{-1} + 1},$$

where

$$\alpha = \frac{\cos\pi(F_1 + F_2)T}{\cos\pi(F_2 - F_1)T},$$

$$k = \cot [\pi(F_2 - F_1)T] \tan(\pi\beta T).$$

By letting  $k = 1$ , then  $\beta = F_2 - F_1$ , and the transformation is simplified.

$$z^{-1} \rightarrow -z^{-1} \left( \frac{z^{-1} - \alpha}{1 - \alpha z^{-1}} \right).$$

The resulting band-pass transfer function has the form

$$H_{BP}(z^{-1}) = (1 - z^{-2}) \sum_{k=1}^{n/2} \left( \frac{A'_{11k}z^{-1} + A'_{01k}}{1 + B'_{11k}z^{-1} + B'_{21k}z^{-2}} - \frac{A'_{12k}z^{-1} - A'_{02k}}{1 - B'_{12k}z^{-1} + B'_{22k}z^{-2}} \right)$$

<sup>1</sup>A. G. Constantinides, *Spectral Transformations for Digital Filters*, Proc. IEE, 117 (August 1970), 1585-1590.

<sup>2</sup>A. Antoniou, *Digital Filters: Analysis and Design*, McGraw-Hill Book Co., Inc., New York (1979).

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for  $n$  even. For  $n$  odd, there is one real pole, and the transfer function is of the form

$$H_{BP}(z^{-1}) = \frac{A_0(1 - z^{-2})}{1 - \alpha(1 - B_1)z^{-1} - B_1z^{-2}}$$

$$+ (1 - z^{-2}) \sum_{k=1}^{(n-1)/2} \left( \frac{A'_{11k}z^{-1} + A'_{01k}}{1 + B'_{11k}z^{-1} + B'_{21k}z^{-2}} \right.$$

$$\left. - \frac{A'_{12k}z^{-1} - A'_{02k}}{1 - B'_{12k}z^{-1} + B'_{22k}z^{-2}} \right).$$

### A-9. BAND-STOP TRANSFORMATION

The procedure for the band-stop transformation is the same as for the band-pass transformation. In general, the transformation is

$$z^{-1} \rightarrow \frac{z^{-2} - \frac{2\alpha}{1+k}z^{-1} + \frac{1-k}{1+k}}{\frac{1-k}{1+k}z^{-2} - \frac{2\alpha}{1+k}z^{-1} + 1},$$

where

$$\alpha = \frac{\cos\pi(F_1 + F_2)T}{\cos\pi(F_2 - F_1)T},$$

$$k = \tan [\pi(F_2 - F_1)T] \tan (\pi\beta T).$$

By letting  $k = 1$ , then  $\beta = F_s/2 - (F_2 - F_1)$ , and the transformation is simplified.

$$z^{-1} \rightarrow z^{-1} \left( \frac{z^{-1} - \alpha}{1 - \alpha z^{-1}} \right).$$

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The resulting band-stop transfer function has the form

$$H_{BS}(z^{-1}) = (1 - 2\alpha z^{-1} + z^{-2}) \sum_{k=1}^{n/2} \left( \frac{A'_{11k} z^{-1} - A'_{01k}}{1 - B'_{11k} z^{-1} + B'_{21k} z^{-2}} \right. \\ \left. - \frac{A'_{12k} z^{-1} + A'_{02k}}{1 + B'_{12k} z^{-1} + B'_{22k} z^{-2}} \right)$$

for n even. For n odd, the transfer function has the form

$$H_{BS}(z^{-1}) = \frac{(1 - 2\alpha z^{-1} + z^{-2})A_0}{1 - \alpha(1 + B_1)z^{-1} + B_1 z^{-2}} \\ + (1 - 2\alpha z^{-1} + z^{-2}) \sum_{k=1}^{(n-1)/2} \left( \frac{A'_{11k} z^{-1} - A'_{01k}}{1 - B'_{11k} z^{-1} + B'_{21k} z^{-2}} \right. \\ \left. - \frac{A'_{12k} z^{-1} + A'_{02k}}{1 + B'_{12k} z^{-1} + B'_{22k} z^{-2}} \right).$$

### A-10. FILTER IMPLEMENTATION

From the z domain general transfer equation

$$H(z^{-1}) = (1 + z^{-1}) \sum_{i=1}^r \frac{A_{0i} + A_{1i} z^{-1}}{1 + B_{1i} z^{-1} + B_{2i} z^{-2}},$$

a time-domain difference equation is obtained. The difference equation is then used to process data samples.

$$Y_{out}(kT) = X(kT) + X[(k - 1)T],$$

## APPENDIX A

where

$$X(kT) = \sum_{i=1}^r X_i(kT),$$

$$X_i(kT) = A_{0i} Y_{in}(kT) + A_{1i} Y_{in}[(k-1)T] \\ - B_{1i} X_i[(k-1)T] - B_{2i} X_i[(k-2)T].$$

APPENDIX B.--FORTRAN IV COMPUTER PROGRAM FOR RECURSIVE DIGITAL FILTERS

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The main program and all of the subroutines for recursive digital filters are included in this appendix. The only exceptions are standard library functions and the plotting routine. If a machine other than an IBM is used, it may be necessary to modify the format and data statements.

## B-1. MAIN PROGRAM

```

REAL*8 A0(20), A1(20), B1(20), B2(20), PI,A2
REAL*8 AMP
COMPLEX*16 P(20), Z(20)
COMMON FC, FS, EPS1, EPS2, PI, ITYPE, N, ITRANS
COMMON/VDATA/VIN(1000), VOUT(1000),NTIMES
COMMON/TRANS/ F1,F2
COMMON/FILT/ WC, W1,W2,AMP,A0,A1,A2, B1, B2, P, Z
C ITYPE- FILTER- BUTTERWORTH(1), CHEBYSHEV(2), ELLIPTIC(3)
C N - ORDER OF FILTER, 20 MAX.
C ITRANS- NONE(0), HP(1), BP(2), BS(3)
C FC- CUTOFF FREQUENCY
C FS- SAMPLE FREQUENCY
C EPS1 - MINIMUM AMPLITUDE IN PASSBAND
C EPS2 - TRANSITION COEFFICIENT FOR CAUER, 0.95 MAX.
C F1 - LOWER CUTOFF FREQUENCY, PASS AND STOP FILTERS ONLY
C F2 - UPPER CUTOFF FREQUENCY, PASS AND STOP FILTERS ONLY
C NTIMES - NUMBER OF DATA POINTS
C VIN- INPUT DATA
C VOUT- OUTPUT DATA
P3=4.0D0*DATA(1.0D0)
CALL RDATA
WC=2*DTAN(PI*FC/FS)
W1=PI*F1/FS
W2=PI*F2/FS
GO TO (10,20,30),ITYPE
10 CALL BUTTER
GO TO 40
20 CALL CHEB
GO TO 40
30 CALL CAUER
40 CALL FACTOR
CALL FPLOT
JJ=ITRANS+1
GO TO (100,50,60,70),JJ
50 CALL HIGHP
GO TO 80

```

APPENDIX B

B-1. MAIN PROGRAM (Cont'd)

```
60  CALL BANDP
70  GO TO 80
70  CALL BANDS
80  CALL FPLOT
100 IF(NTIMES.EQ.0)GO TO 110
     CALL FILIMP
     CALL PLTDAT
110 CONTINUE
     STOP
     END
```

B-2. SUBROUTINE RDATA

```
SUBROUTINE RDATA
REAL#8 PI
COMMON FC, FS, EPS1, EPS2, PI, ITYPE, N, ITRANS
COMMON/VDATA/VIN(1000), VOUT(1000), NTIMES
COMMON/TRANS/ F1,F2
READ(5,10) ITYPE, N, ITRANS, FC,FS,EPS1, EPS2
10  FORMAT(3I5, 4E10.3)
F2=0
F1=F2
IF (ITRANS.EQ.0) GO TO 100
READ(5,20) F1,F2
20  FORMAT(2E10.3)
READ(5,25)NTIMES
25  FORMAT(15)
IF(NTIMES.EQ.0)GO TO 50
JJ=(NTIMES+7)/8
DO 100 K=1,JJ
L=(K-1)*8+1
M=K*8
100 READ(5,30)(VIN(I),I=L,M)
30  FORMAT(8E10.3)
50  CONTINUE
      WRITE(6,45)
45  FORMAT(1//,5X5HITYPE,7X1HN,6X6HITRANS,9X2HFC,13X2HFS,12X4HEPS1,
111X4HEPS2,12X2HF1,13X2HF2)
      WRITE(6,40) ITYPE, N, ITRANS, FC, FS, EPS1, EPS2, F1, F2
40  FORMAT(1,3(5X15), 6(5XE10.3),1///)
      RETURN
      END
```

APPENDIX B

B-3. SUBROUTINE BUTTER

```
SUBROUTINE BUTTER
REAL*8 A0(20), A1(20), B1(20), B2(20), P1,PHI
COMPLEX*16 P(20), Z(20)
COMMON FC, FS, EPS1, EPS2, P1, ITYPE, N, ITRANS
COMMON/FILT/WC, W1,W2,AMP,A0,A1,A2, B1, B2, P, Z
REAL*8 AMP,A2
A=N
K=N-2*INT(N/2.)
DO 20 I=1,N
B=J
IF(K.EQ.0) GO TO 5
PHI=P1*(A+1)/2.0+B-1.0)/A
GO TO 10
5 PHI=P1*(1.0/(2.0*A)+(B-1.0)/A+0.5)
P11=WC*DCMPLX(DCOS(PHI),DSIN(PHI))
CONTINUE
AMP=WC**N
RETURN
END
```

B-4. SUBROUTINE FACTOR

```
SUBROUTINE FACTOR
REAL*8 AMP
REAL*8 A0(2), A1(20), B1(20), B2(20), P1, RR,R1,D, AK, BK,A2
COMPLEX*16 P(20), Z(20), R(20), X
COMMON FC, FS, EPS1, EPS2, P1, ITYPE, N, ITRANS
COMMON/FILT/WC, W1,W2,AMP,A0,A1,A2, B1, B2, P, Z
A2=1.0D0
K= INT((N+1)/2.)
DO 50 I=1,K
X=DCMPLX(1.0D0, 0.0D0)
DO 20 J=1,N
IF (J.EQ.I) GO TO 20
X=X*(P(I)-P(J))
20 CONTINUE
R(I)= AMP/X
IF(N.EQ.1)GO TO 30
X=DCMPLX(1.0D0,0.0D0)
IF(ITYPE.NE.3)GO TO 50
KK=2*(N/2)
DO 30 J=1,KK
X=X*(P(I)-Z(J))
30
```

## APPENDIX B

## B-4. SUBROUTINE FACTOR (Cont'd)

```

30  CONTINUE
    R(I)=R(I)*X
    AMP=AMP*(KK+1-N)
50  CONTINUE
    IF(I TYPE .NE. 3)AMP=0.000
    DO 100 I=1,K
    RR=DREAL(R(I))
    RI=DIMAG(R(I))
    AK=-DREAL(P(I))+0.500
    BK= DIMAG(P(I))+0.500
    D=(1.000+AK)**2+BK**2
    A0(I)=(RR*(1.000+AK)-RI*BK)/D
    A1(I)=-(RR*(1.000-AK)+RI*BK)/D
    B1(I)= -2.000*(1.000-AK**2-BK**2)/D
    B2(I)=(1.000-AK)**2+BK**2)/D
100  CONTINUE
    I=2*K-N
    IF(I.EQ.0)GO TO 110
    A0(K)=A0(K)/2
    B1(K)=B1(K)/2
    A1(K)=0.
    B2(K)=0.
110  RETURN
    END

```

## B-5. SUBROUTINE CHEB

```

SUBROUTINE CHEB
COMMON/FILT/WC, N1, N2, AMP, A0, A1, A2, B1, B2, P, Z
COMMON FC, FS, EPS1, EPS2, PI, ITYPE, N, ITRANS
COMPLEX*16 P(20), Z(20)
REAL*8 A0(20), A1(20), B1(20), B2(20)
REAL*8 CHV, SHV, V, J, AMP, PI, A2
EPS1=SQRT(1.0/EPS1**2-1.0)
A2=1.000
A=N
V=DLOG(1.000/EPS1+DSQRT(1.000*1.000/EPS1**2))/A
CHV=(DEXP(V)+DEXP(-V))/2.000
SHV=(DEXP(V)-DEXP(-V))/2.000
JJ=(N+1)/2
DO 50 I=1,N
U=P1*(2.000*(I+N)-1.000)/(2.000*A)
P(I)=WC*D(MPLX(DSIN(U)*SHV, -DCOS(U)*CHV)

```

## APPENDIX B

## B-5. SUBROUTINE CHEB (Cont'd)

```

50  CONTINUE
K=N-2*JJ
AMP=1.000
IF(K.EQ.0)GO TO 90
JJ=JJ-1
IF(N.EQ.1)GO TO 80
DO 75 I=1,JJ
AMP=AMP*(DREAL(P(I))**2+DIMAG(P(I))**2)
75  CONTINUE
80  AMP=AMP*DREAL(P(JJ+1))
GO TO 125
90  DO 100 I=1,JJ
AMP=AMP*(DREAL(P(I))**2+DIMAG(P(I))**2)
100 CONTINUE
AMP=AMP/DSQRT(1.000*EPS1**2)
125 RETURN
END

```

## B-6. SUBROUTINE CAUER

```

SUBROUTINE CAUER
COMMON/FILT/WC, W1, W2, AMP, A0, A1, A2, B1, B2, P, Z
COMMON FC, FS, EPS1, EPS2, PI, ITYPE, N, ITRANS
REAL*8 AMP, Q, A, S0, W, O, V
REAL*8 A0(20), A1(20), B1(20), B2(20), PI, A2
COMPLEX*16 Z(20), P(20)
S0=0.000
W=0.000
Q=(1.000-DBLE(EPS2)**2)**(0.25)
C=0.500*(1.000-Q)/(1.000+Q)
AMP=1.000
Q=Q+2.0*Q**5+15.0*Q**9+150.0*Q**13
A=DBLE(EPS1)
A=DLG((1.000/A+1.000)/(1.000/A-1.000))/(2.0*N)
O=0.000
DO 10 I=1,100
M=I-1
S0=S0+(-1.000)**M*Q**((M+1)*I)*DSINH((I+M)*A)
IF(W.EQ.S0)GO TO 20
10 W=S0
20 DO 30 I=1,100
O=O+(-1.000)**I*Q**((I+1)*I)*DCOSH(2.0*I*A)
IF(O.EQ.W)GO TO 40
30 W=0

```

## APPENDIX B

## B-6. SUBROUTINE CAUER (Cont'd)

```

40      S0=DABS(2.0*Q**10.25)*S0/(1.000*2.0*0)
C=N
W=DSQRT((1.0D0+EPS2*S0**2)*(1.000+S0**2/EPS2))
IF(N.EQ.1)GO TO 100
JJ=N/2
U=(2*JJ+1-N)/2.0
DO 100 I=1,JJ
D=0.000
V=0
DO 50 J=1,100,2
M=J-1
D=(-1.0)**M*Q**((M+J)*DSIN((M+J)*PI*(FLOAT(I)-U)/C)+0
M=J+1
D=(-1.0)**J*Q**((M+J)*DSIN((M+J)*PI*(FLOAT(I)-U)/C)+0
IF(0.EQ.V)GO TO 60
50      V=0
60      A=0.000
V=A
DO 70 J=1,100,2
M=J
V=V+(-1.0)**M*Q**((M+M)*DCOS(2.0*M*PI*(FLOAT(I)-U)/C)
M=J+1
V=V+(-1.0)**M*Q**((M+M)*DCOS(2.0*M*PI*(FLOAT(I)-U)/C)
IF(V.EQ.A)GO TO 80
70      A=V
80      D=2.0*Q**((0.25)*0/(1.000*2.0*V)
V=(1.000-EPS2*0*0)*(1.000-0*0/EPS2)
V=DSQRT(V)
Z(1)=DCMPLX(0.0D0,1.000/0)*WC
Z(N+1-1)=DCONJG(Z(1))
A=1.000+(S0*0)**2
P(1)=DCMPLX(-S0*V/A,0*W/A)*WC
P(N+1-1)=DCONJG(P(1))
AMP=AMP+0*0*((S0*V)**2+(0*W)**2)/A**2
100    CONTINUE
      K=N-2*(N/2)
IF(K.EQ.0)GO TO 150
P((N+1)/2)=DCMPLX(-S0*WC,0.000)
AMP=AMP+S0*WC
GO TO 200
150    AMP=AMP*EPS1
200    CONTINUE
      RETURN
      END

```

## APPENDIX B

## B-7. SUBROUTINE HIGHP

```

SUBROUTINE HIGHP
REAL*8 A0(2), A1(20), B1(20), B2(20), PI
REAL*8 AMP,A2
REAL*8 AP,A00,A11,B11,B22,D
COMMON FC, FS, EPS1, EPS2, P, ITYPE, N, ITRANS
COMMON/FILT/HC, W1,W2,AMP,A0,A1,A2, B1, B2, P, Z
W3=PI*FC/FS
A2=-1.000
AP=-COS(W1+W3)/COS(W1-W3)
DO 100 K=1,N
D=1.000-B1(K)*AP+B2(K)*AP*AP
A00=(A0(K)-A1(K)*AP)*(1.000-AP)/D
A11=(A0(K)*AP-A1(K))*(1.000-AP)/D
B11=(AP*AP-B1(K)*(1.000*AP*AP)+2.000*B2(K)*AP)/D
B22=(AP*AP-AP*B1(K)+B2(K))/D
A0(K)=A00
A1(K)=A11
B1(K)=B11
B2(K)=B22
100  CONTINUE
RETURN
END

```

## B-8. SUBROUTINE FPLOT

```

SUBROUTINE FPLOT
COMMON FC, FS, EPS1, EPS2, P, ITYPE, N, ITRANS
COMMON/FILT/HC, W1,W2,AMP,A0,A1,A2, B1, B2, P, Z
REAL*8 AMP
REAL*8 A0(20), A1(20), B1(20), B2(20), PI,A2
DIMENSION F(251),HE3WT(251),PHI(251)
DIMENSION XLAB(2),YLAB1(2),YLAB2(2),HEAD(10),SUBH(2)
DATA YLAB2(1),YLAB2(2)/8PHASE 1D,BMEG/
DATA YLAB1(1),YLAB1(2)/8HAMPL3TUD,BHE/
DATA XLAB(1),XLAB(2)/8HFREQUENC,BHY 8F/FS)/
WRITE(6,800)
JJ=(N+1)/2
DO 805 I=1,JJ
805 WRITE(6,801)A0(I),A1(I),B1(I),B2(I)
WRITE(6,802)AMP,A2
800 FORMAT(16X2HA1,32X2HA1,32X2HB1,32X2HB2,///)
801 FORMAT(1P4(1XQ32.25))

```

## APPENDIX B

## B-8. SUBROUTINE FPLOT (Cont'd)

```

802  FORMAT(4X6HA4P = ,1P032.25,10X5HA2 = ,0PF4.0)
JJ=(N+1)/2
READ(5,10) (HEAD(I),I=1,10)
10  FORMAT(10A8)
DO 100 I=1,250
A=I-1
F(I)=A/249.0
W=2.0*PI*F(I)
C=COS(W)
S=SIN(W)
PH1(I)=0.
HEJNT(I)=0.
DO 50 K=1,JJ
CN=A0(K)+A1(K)*C
DN=-A1(K)*S
CD=1.0+B1(K)*C+32(K)*(2.0*C-1.0)
DD=B1(K)*S+2.0*32(K)*C*S
DD=-DD
EE=CD*CD+DD*DD
FF=(CN*(CD+DN*DD))/EE
EE=(DN*(CD-DD*CN))/EE
HEJNT(I)=HEJNT(I)+FF
PH1(I)=PH1(I)+EE
50  CONTINUE
FF=(1.0D0*A2*C)*HEJNT(I)+S*PH1(I)*A2      +AMP
EE=(1.0D0*A2*C)*PH1(I)-S*HEJNT(I)*A2
HEJNT(I)=SQRT(EE*EE+FF*FF)
PH1(I)=ATAN2(EE,FF)
PH1(I)=PH1(I)*180.0/PI
100 CONTINUE
CALL DRAWID(1,4,4,20,0,250,2.,0.,XLAB,
1           YLAB1,HEAD,SUBH,F,HEJNT)
F(I)=0.0
CALL DRAWID(1,4,4,20,0,250,2.0,0.,XLAB,
1YLAB2,HEAD,SUBH,F,PH1)
RETURN
END

```

```

SUBROUTINE BANDP
COMMON/FILT/W1,W2,AMP,A0,A1,A2, B1, B2, P, Z
COMMON FC, FS, EPS1, EPS2, PI, ITYPE, N, ITRANS
REAL*B A0(20), A1(20), B1(20), B2(20), PI,A2,AMP
COMPLEX*36 Z(20),P(20),PP,Q,R,S,T,U,V
REAL*B C,D,AL,E,F
A2=0.000
AL=DCOS(DBLE(W1+W2))/DCOS(DBLE(W2-W1))
JJ=N/2
IF(N.EQ.1)GO TO 50
DO 50 I=1,JJ
AMP=AMP+A0(1)
C=B2(1)-B1(1)*2/4.000
IF(C.LE.0.000)GO TO 40
PP=DCMPLX(-B1(1)/2.000,DSQRT(8C))
Q=(A0(1))*PP+PP+(A1(1))*PP+DCMPLX(A1(1),0.000))
1/DCMPLX(0.000,2.000*DIMAG(PP))
V=DCMPLX(1.000,0.000)
S=AL*(V+PP)/2.000
R=S+CDSQRT(S*S-PP)
S=S-CDSQRT(S*S-PP)
T=0*(AL+R-V)/(R-S)
U=Q*(AL+S-V)/(S-R)
C=DREAL(R)
D=DIMAG(R)
E=DREAL(T)
F=DIMAG(T)
F=-2.0*(C*E+D*F)
D=C*C*D*D
C=-2.0*C
E=2.0*E
AMP=AMP+F/D
A0(1)=-F/D
A1(1)=E-F*C/D
B1(1)=C
B2(1)=D
K=(N+1)/2+1
C=DREAL(S)
D=DIMAG(S)
E=DREAL(U)
F=DIMAG(U)
F=-2.0*(C*E+D*F)
D=C*C*D*D
C=-2.0*C
E=2.0*E
AMP=AMP+F/D
A0(K)=-F/D
A1(K)=E-F*C/D
B1(K)=C
B2(K)=D
GO TO 50

```

APPENDIX B

B-9. SUBROUTINE BANDP (Cont'd)

```

40    C=DSQRT(1-C)
      D=-B1(1)/2.000*C
      C=-B1(1)/2.000-C
      E=(AO(1)*C+(AO(1)*A1(1))*C+A1(1)*C)/8(C-D)
      F=(AO(1)*D+D*(AO(1)*A1(1))*D+A1(1)*D)/8D-C)
      AMP=AMP-E/C-F/D
      AO(1)=E/C
      A1(1)=-AL*E/C
      B1(1)=-AL*(1.0D)+C
      B2(1)=C
      K=(N+1)/2+1
      AO(K)=F/D
      A1(K)=-AL*F/D
      B1(K)=-AL*(1.0D)+D
      B2(K)=D
50    CONTINUE
      K=N-2*JJ
      IF(K.EQ.0)GO TO 100
      AMP=AMP+AO(N)/B1(N)
      D=B1(N)
      C=AO(N)*(1.0D-1.0/D)
      K=(N+1)/2
      AO(K)=C
      A1(K)=-AL*C
      B1(K)=AL*(D-1.0D)
      B2(K)=-D
100   N=2*N
      RETURN
      END

```

B-10. SUBROUTINE BANDS

```

SUBROUTINE BANDS
COMMON/FILT/W1,W2,AMP,A0,A1,A2, B1, B2, P, Z
COMMON FC, FS, EPS1, EPS2, PI, ITYPE, N, ITRANS
REAL*8 AO(20), A1(20), B1(20), B2(20), PI,A2,AMP
COMPLEX*16 Z(20),P(20),PP,Q,R,S,T,U,V
REAL*8 C,D,AL,E,F,X,C1,C2
A2=0.000
AL=DCOS(DBLE(W1+W2))/DCOS(DBLE(W2-W1))
JJ=N/2
X=DTAN(DBLE(W2-W1))*DTAN(PI)*FC/FS
C1=2.0*AL/(X+1.0D0)
C2=(1.0D0-X)/(X+1.0D0)

```

## APPENDIX B

## B-10. SUBROUTINE BANDS (Cont'd)

```

IF(N.EQ.3)GO TO 50
DO 50 I=1,JJ
V=DCMPLX(1.0D0,0.0D0)
AMP=AMP+AD(I)
C=B2(I)-B1(I)**2/4.0D0
IF(C.LE.0.0D0)GO TO 40
PP=DCMPLX(-B1(I)/2.0D0,DSQRT(K))
Q=(AD(I)*PP+PP*(AD(I)+A1(I)))*PP+DCMPLX(A1(I),0.0D0)
I/DCMPLX(0.0D0,2.0D0*DIMAG(PP))
Q=Q+C2/(V-PP+C2)
AMP=AMP+2.0*DREAL(Q)
S=(1+(V-PP)/(V-C2*PP))**2.0
T=(C2*V-PP)/(V-C2*PP)
R=S+CDSQRT(S+S-T)
S=S-CDSQRT(S+S-T)
T=Q*(R+S-C1/C2*S+V/C2)/(S-R)
U=Q*(S+S-C1/C2*S+V/C2)/(S-R)
C=DREAL(R)
D=DIMAG(R)
E=DREAL(T)
F=DIMAG(T)
F=-2.0*(C+E+D+F)
D=C+C*D*D
C=-2.0*C
E=2.0*E
AMP=AMP+F/D
AD(I)=-F/D
A1(I)=E-F*C/D
B1(I)=C
B2(I)=D
K=(N+1)/2+1
C=DREAL(S)
D=DIMAG(S)
E=DREAL(U)
F=DIMAG(U)
F=-2.0*(C+E+D+F)
D=C+C*D*D
C=-2.0*C
E=2.0*E
AMP=AMP+F/D
AD(K)=-F/D
A1(K)=E-F*C/D
B1(K)=C
B2(K)=D
GO TO 50

```

## APPENDIX B

## B-10. SUBROUTINE BANDS (Cont'd)

```

40  C=DSQRT(E-C)
  D=-B1(I)/2.0D0+C
  C=-B1(I)/2.0D0-C
  E=(A0(I)*C+C*(A1(I)+A1(I))*C+A1(I))/((C-D)
  F=(A0(I)*D*D+(A1(I)+A1(I))*D+A1(I))/((D-C)
  AMP=AMP+E/((C2-C)+F/((C2-D)
  A0(I)=E*(C2/(1.0D0+C2)-1.0/(C2-C))
  A1(I)=-E*(1*(C+C2)/(1.0D0+C2)**2
  B1(I)=C1*(C-1.0D0)/(1.0D0+C2)
  B2(I)=(C2-C)/(1.0D0+C2)
  K=I+N
  B2(K)=(C2-D)/(1.0D0+C2)
  B1(K)=C1*(D-1.0D0)/(1.0D0+C2)
  A0(K)=F*(C2/(1.0D0+C2)-1.0/(C2-D))
  A1(K)=-F*(1*(D+C2)/(1.0D0+C2)**2
50  CONTINUE
  K=N-2*JJ
  IF(K.EQ.0) GO TO 100
  K=(N+1)/2
  AMP=AMP+A0(K)
  D=B1(K)
  C=A0(K)/(1.0D0+B1(K)*C2)*(1.0D0-D)
  B2(K)=(C2+D)/(1.0D0+C2*D)
  B1(K)=-C1*(D+1.0D0)/(1.0D0+C2*D)
  AMP=AMP+C/B2(K)
  A0(K)=C+C2-C/B2(K)
  A1(K)=-C1*(C-B1(K))/B2(K)
100 N=2*NN
  RETURN
  END

```

## B-11. SUBROUTINE FILIMP

```

SUBROUTINE FILIMP
COMMON FC, FS, EPS1, EPS2, P1, ITYPE, N, ITRANS
REAL*8 AMP
REAL*8 A0(20), A1(20), B1(20), B2(20), PI,A2
COMMON/VDATA/ VIN(1000),VOUT(1000),NTIMES
COMMON/FILT/WC, W1,W2,AMP,A0,A1,A2, B1, B2, P, Z
REAL*8 YK(2,20), YSUM1, YSUM2, YTEM
YSUM1=0.0
YSUM2=0.0
JJ=(N+1)/2
DO 10 K=1, JJ

```

## APPENDIX B

## B-11. SUBROUTINE FILIMP (Cont'd)

```

YK(2,K)=AO(K)*VIN(1)
YK(1,K)=AO(K)*VIN(2)*(A1(K)-B1(K)*AO(K))+VIN(1)
YSUM2=YSUM2+YK(2,K)
YSUM1=YSUM1+YK(1,K)
10  CONTINUE
VOUT(1)=YSUM2
VOUT(2)=YSUM1
DO 50 J=3,NTIMES
YSUM2=YSUM1
YSUM1=0.0
DO 40 K=3, JJ
YTEM=AO(K)*VIN(J)+A1(K)*VIN(J-1)
1-B1(K)*YK(1,K)-B2(K)*YK(2,K)
YK(2,K)=YK(1,K)
YK(1,K)=YTEM
YSUM1=YSUM1+YTEM
40  CONTINUE
VOUT(J)=YSUM1+YSUM2*A2      +AMP*VIN(J)
50  CONTINUE
RETURN
END

```

## B-12. SUBROUTINE PLTDAT

```

SUBROUTINE PLTDAT
COMMON/VDATA/VIN(1000),VOUT(1000),NTIMES
COMMON FC,FS
DIMENSION HEAD(10),XLAB(2),YLAB(2),T(1000)
DATA XLAB(1),XLAB(2)/8HTIME (SE,BHCOND) /
DATA SUB1,SUB2/8HVIN      ,8HVOUT      /
DATA YLAB(1),YLAB(2)/8AMPLITUD,BHE      /
DO 20 I=1,NTIMES
20  T(I)=(I-1)/FS
READ(5,10)(HEAD(I),I=1,10)
10  FORMAT(10A8)
CALL DRAW1D(1,4,4,20,2,NTIMES,2.,0.,XLAB,YLAB,HEAD,SUB1,T,VIN)
READ(5,10)(HEAD(I),I=1,10)
CALL DRAW1D(1,4,4,20,2,NTIMES,2.,0.,XLAB,YLAB,HEAD,SUB2,T,VOUT)
RETURN
END

```

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